II. Quantum Many Body Theory for Bose-Einstein Condensate State and Application to LENR

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   c. Optical Theorem Formulation of Nuclear Fusion Reaction
   d. Parameterization of the Short Range Nuclear Force
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II. 1. BEC for Atomic Case
Brief History of Bose-Einstein Condensation (BEC)

1924	Bose: photons
1924	Einstein: atoms and molecules, with integer spins
1932	Specific heat of liquid He II – Keesom and Claussius
1938	Bose-Einstein Condensate Properties of He II Fountain Effect Experiments of Allen and Jones
1990~1995	Development of Magnetic Trapping and Optical Cooling
1995	Development of Evaporation Cooling BEC observed for diluted gases (Rb,Na,Li) at ~100 nano Kelvin (10^{-7} ^{o}K)
1995~present	Extensive studies of properties of BEC and new applications
1999	Hydrogen (H) BEC observed
2000	Metastable Helium (He*) BEC observed
2004	Superfluidity in Solid ^{4}He observed

Table 1. Bosons under study

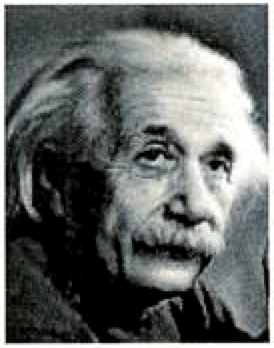
Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	Metals	superconductivity
Cooper pair	h^+, h^+	Copper oxides	High-T_c superconductivity
Exciton	e^-, h^+	Semiconductors	Luminescence and drag-free transport in Cu_2O
Biexciton	$2(e^-, h^+)$	Semiconductors	Luminescence and optical phase coherence in CuC
Positronium	e^-, e^+	Crystal vacancies	(proposed)
Hydrogen	e^-, p^+	Magnetic traps	(BEC)
^4He	$^4He^{2+}$, $2e^-$	He-II, He solid	Superfluidity
^3He pairs	$2(^3He^{2+}, 2e^-)$	^3He-A,B phases	Superfluidity
Cesium	$^{133}_{55}Cs^{55+}, 55e^-$	Laser traps	(in progress)
Rubidium	$^{87}_{39}Rb^{37+}, 37e^-$	Laser & Magnetic Trap	BEC in dilute gas
Lithium	$^{7}_{3}Li^{3+}, 3e^-$	"	"
Sodium	$^{23}_{11}Na^{11+}, 11e^-$	"	"

Table 2. Bosons under study

Particle	Composed of	In	Coherence seen in
Interacting bosons	nn or pp	Nuclei	Excitations
Nucleonic pairing	nn or pp	Nuclei Neutron stars	Moments of inertia superfluidity and pulsar glitches
Chiral condensates	$\langle \bar{q} q \rangle$	Vacuum	Elementary particle structure
Meson condensates	$pion\ condensate = \langle \bar{u} d \rangle, etc.$ $kaon\ condensate = \langle \bar{u} s \rangle$	Neutron star matter	Neutron stars, Supernovae (proposed)
Higgs boson	$\langle \bar{t} t \rangle\ condensate$ $(proposed)$	Vacuum	Elementary particle masses
Nuclei	Protons & neutrons	Condensed matters	Micro- and nano-scale atomic clusters, bubbles and cavities (proposed)

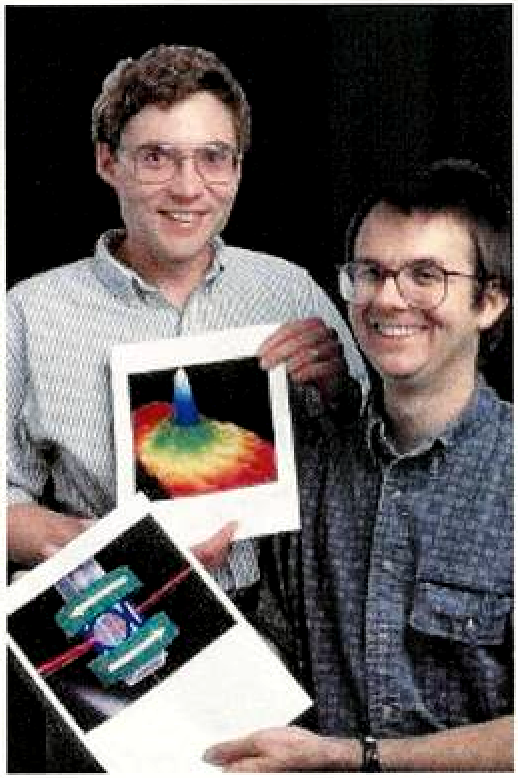
Bose-Einstein Condensation in a gas: a new form of matter at the coldest temperatures in the universe...

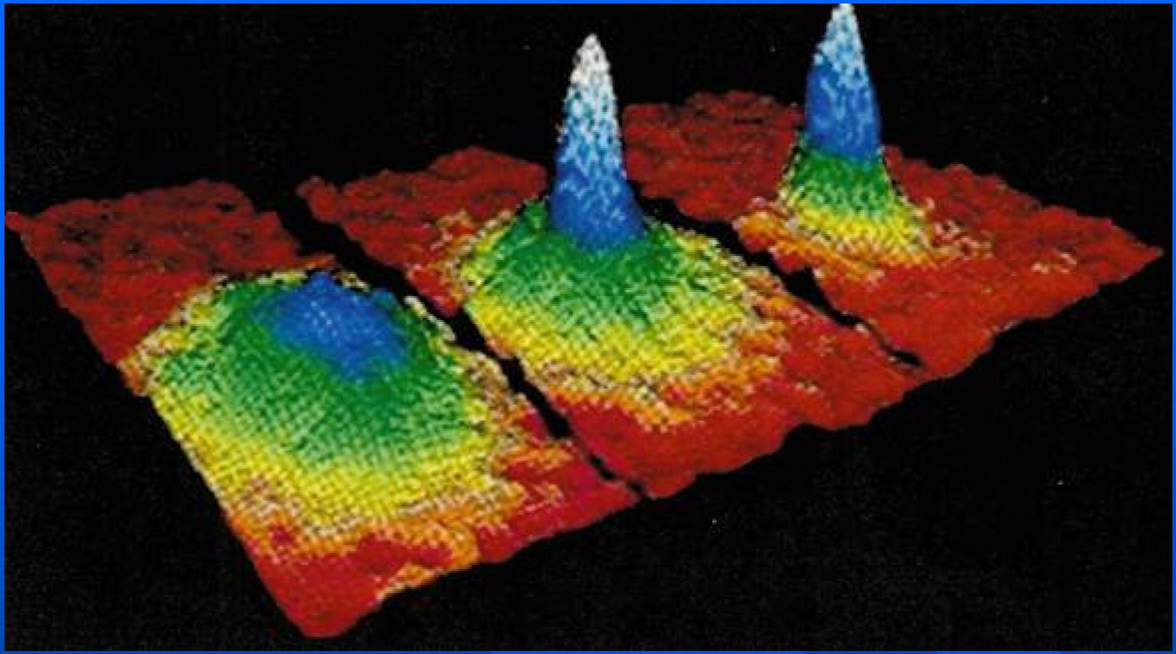
Predicted 1924... ...Created 1995



A. Einstein

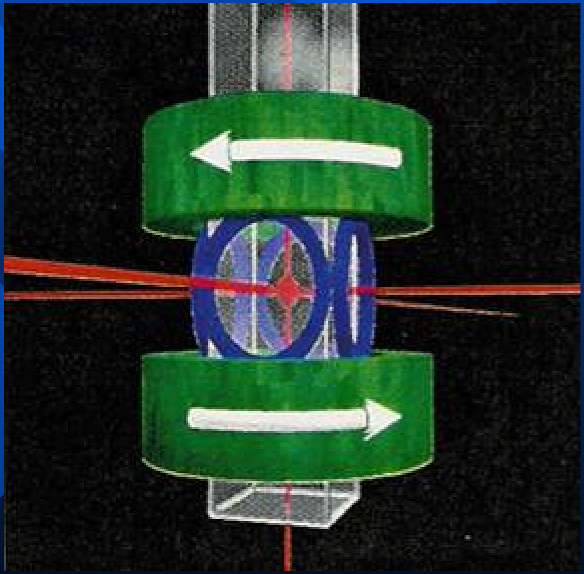
S. Bose

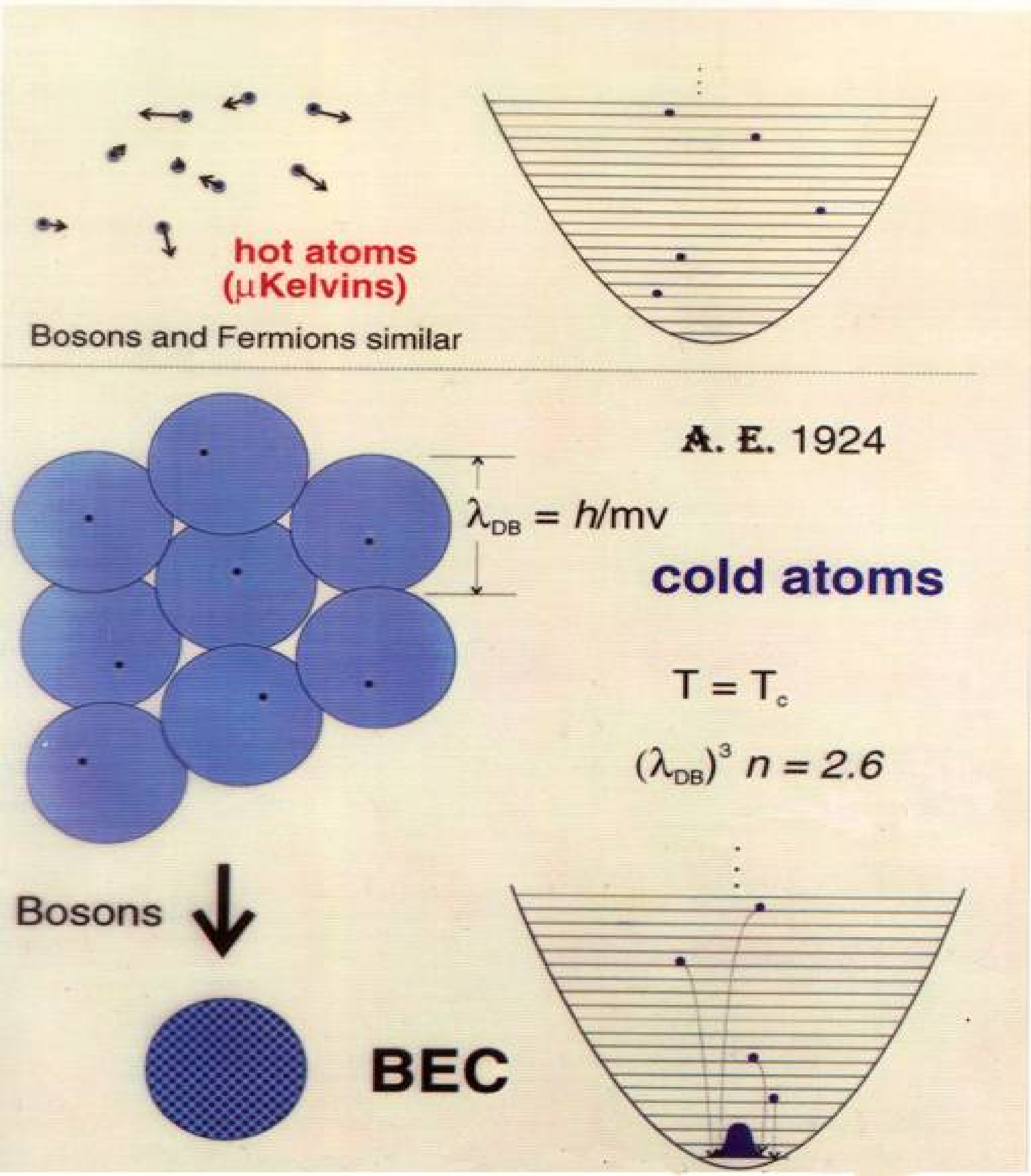




Plots of experimental data for velocity distribution of Rb atoms in BEC as a function of temperatures

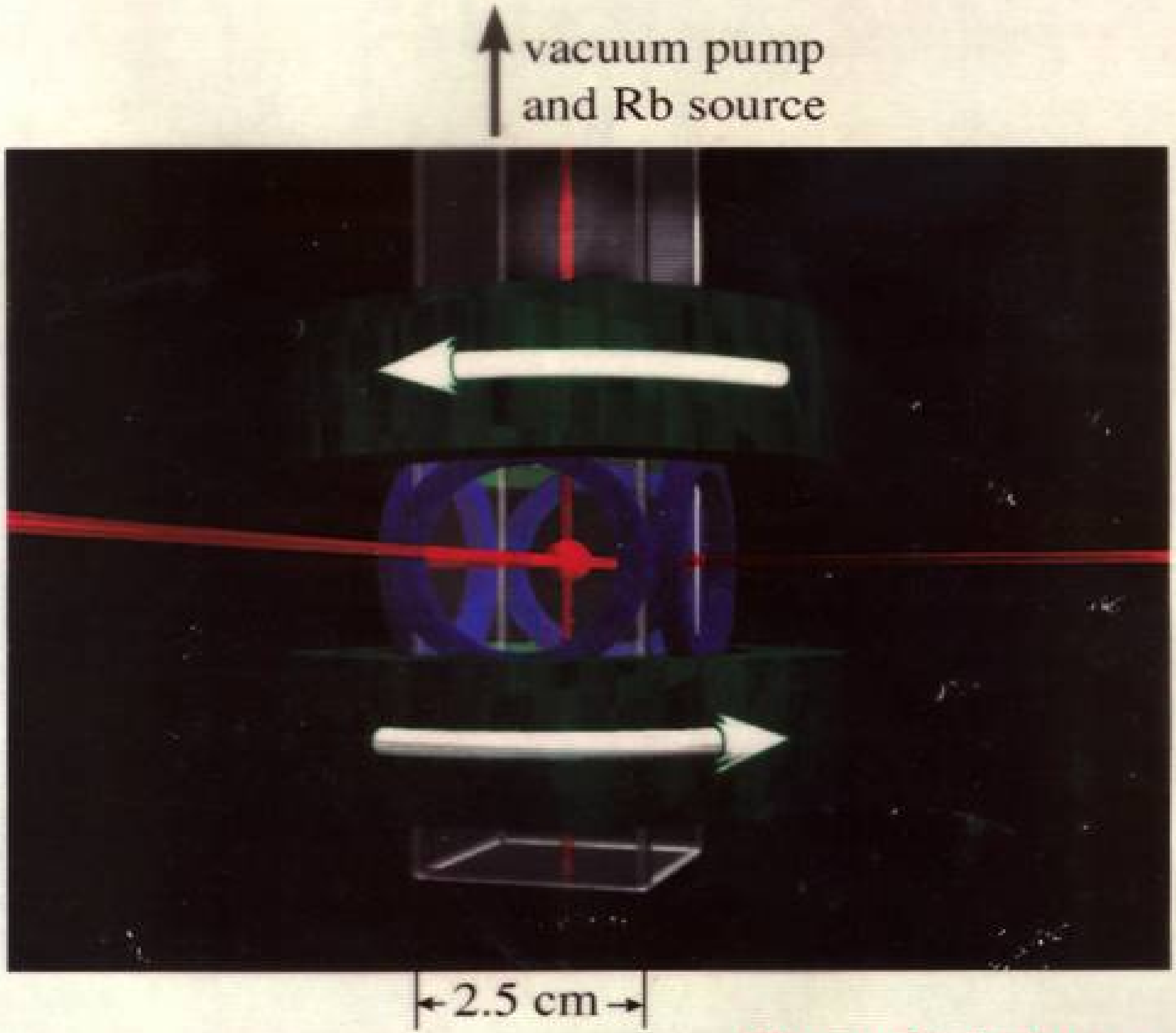
Schematic Picture of BEC Apparatus




hot atoms
(μKelvins)
Bosons and Fermions similar
A. E. 1924
$\lambda_{DB} = h/mv$
cold atoms
$T = T_c$
$(\lambda_{DB})^3 n = 2.6$
Bosons
BEC

BEC Apparatus

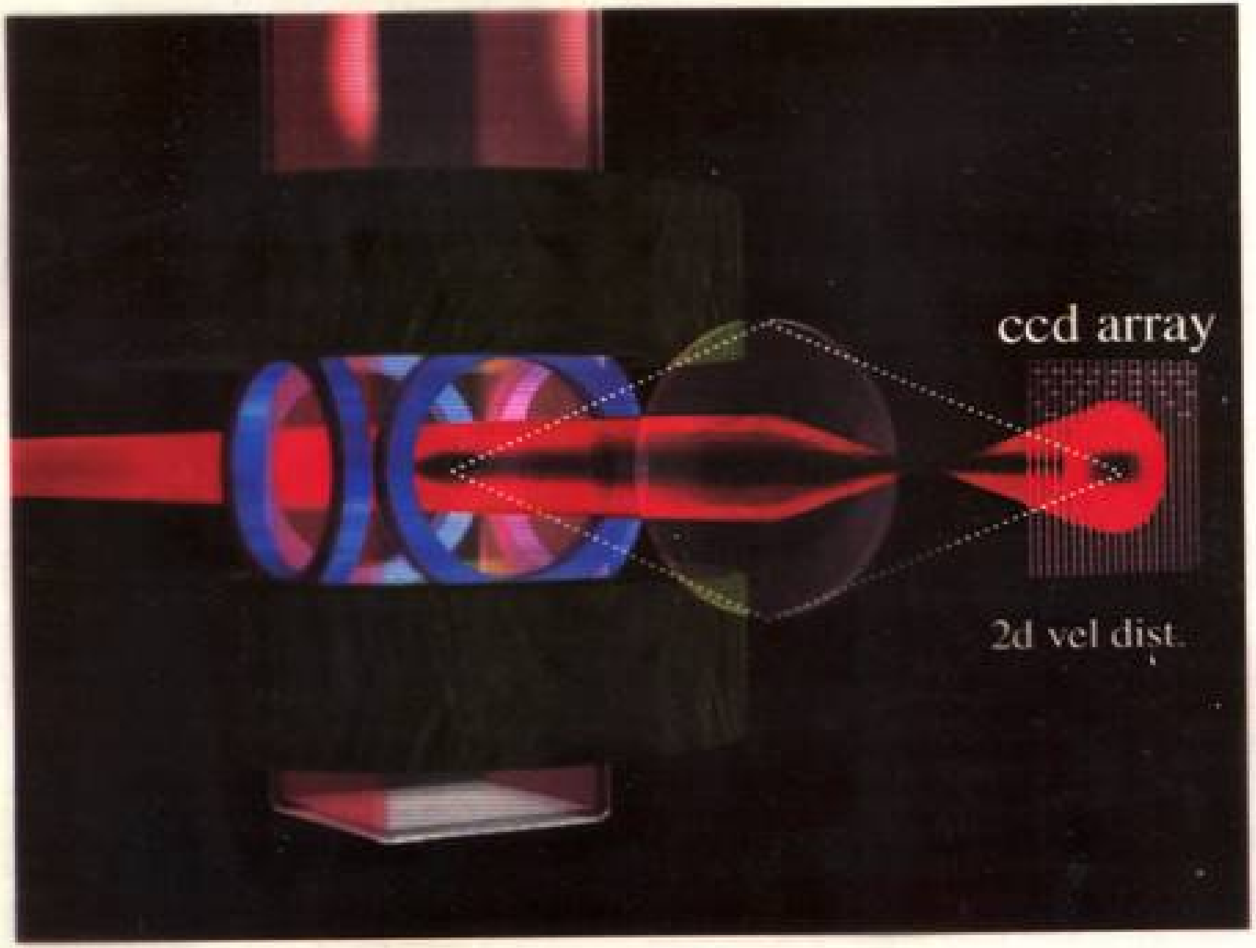
"Jila I" ~ 1/95- 11/96 (RIP Smithsonian)



Cheap diode lasers

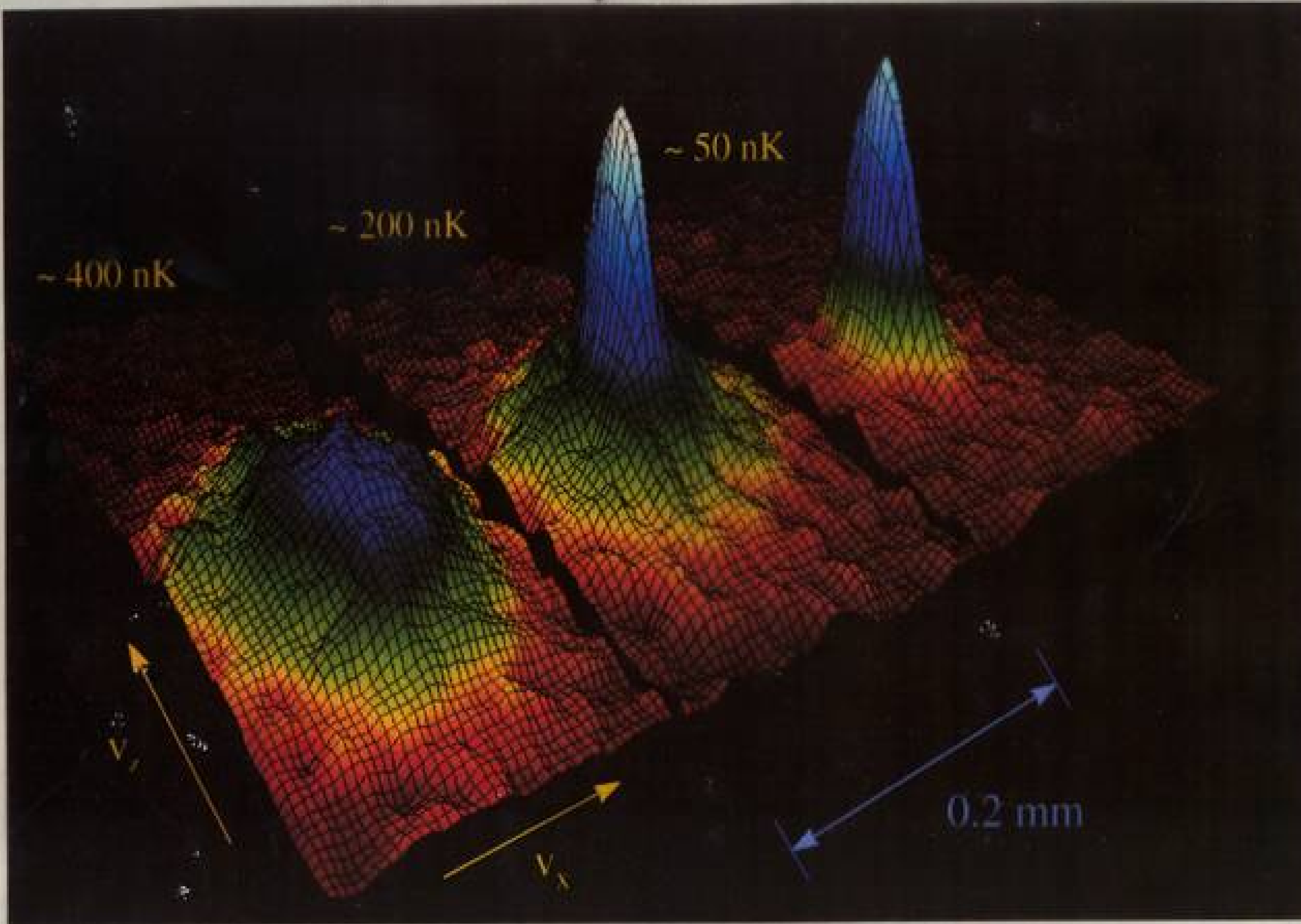
Observing condensate

1. Expand cloud.
2. "Shadow snapshot".



Destroys condensate

2 D velocity distributions



Fundamental Definition of Temperature For Micro Systems

- Temperatures for micro-scale systems are determined from the measurements of velocities using the Bose-Einstein distribution:

$$n(E) = \frac{1}{e^{\alpha} e^{E/kT} - 1} \rightarrow e^{-\alpha} e^{-E/kT}, \; E >> kT$$

- **Bose- Einstein Condensation (BEC) Mechanism for Low Energy Nuclear Reaction**
  (Kim and Zubarev, Fusion Technology **37,** 151 (2000))

II. 2. Motivation for BEC Mechanism

- Induced Fusion with Atomic Clusters (Arata-Zhang Electrolysis Data) (Clarke, Oliver, McKube, Tanzella, and Tripodi, Fusion Science and Technology **40**, 152 (2001))

  No concrete evidence of nuclear products except tritium.
  Tritium production rate of $2x10^8$/sec.
  This rate will produce power at ~100 μ Watts ≈ 0.1m Watts, if we assume D(d,p)T, Q=4.03 Mev.

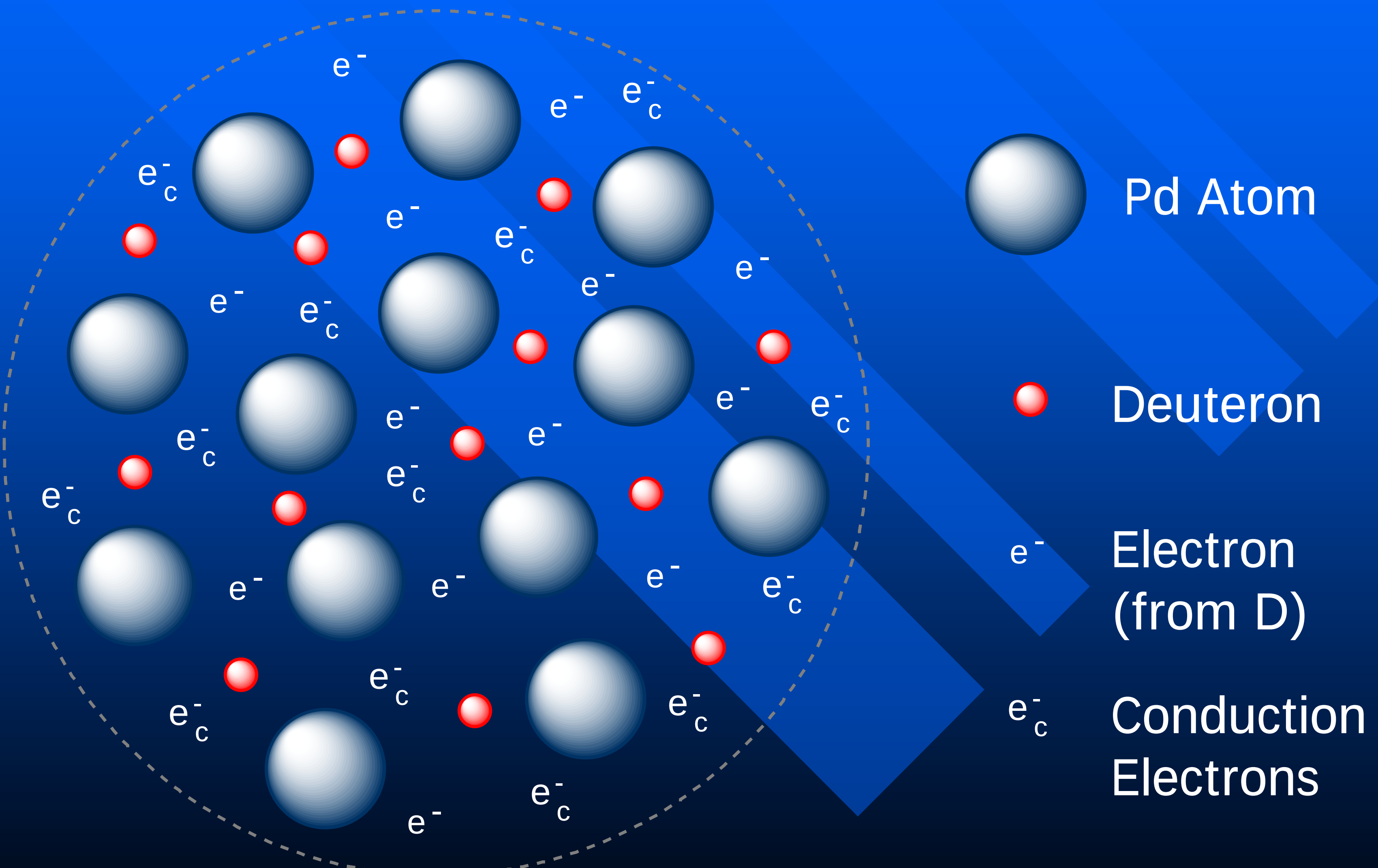
- Earlier sonoluminescence fusion with bubbles indicates some excess heat (Stringham et at., Proceedings of the ICCF - 7, Vancouver, BC, Canada, April 19, 1998, pp. 361-365)

- Sonoluminescence Fusion with Bubbles
  (Taleyarkhan et al., Science **295**, 1868 (2002); Phys. Rev. E, 2004)

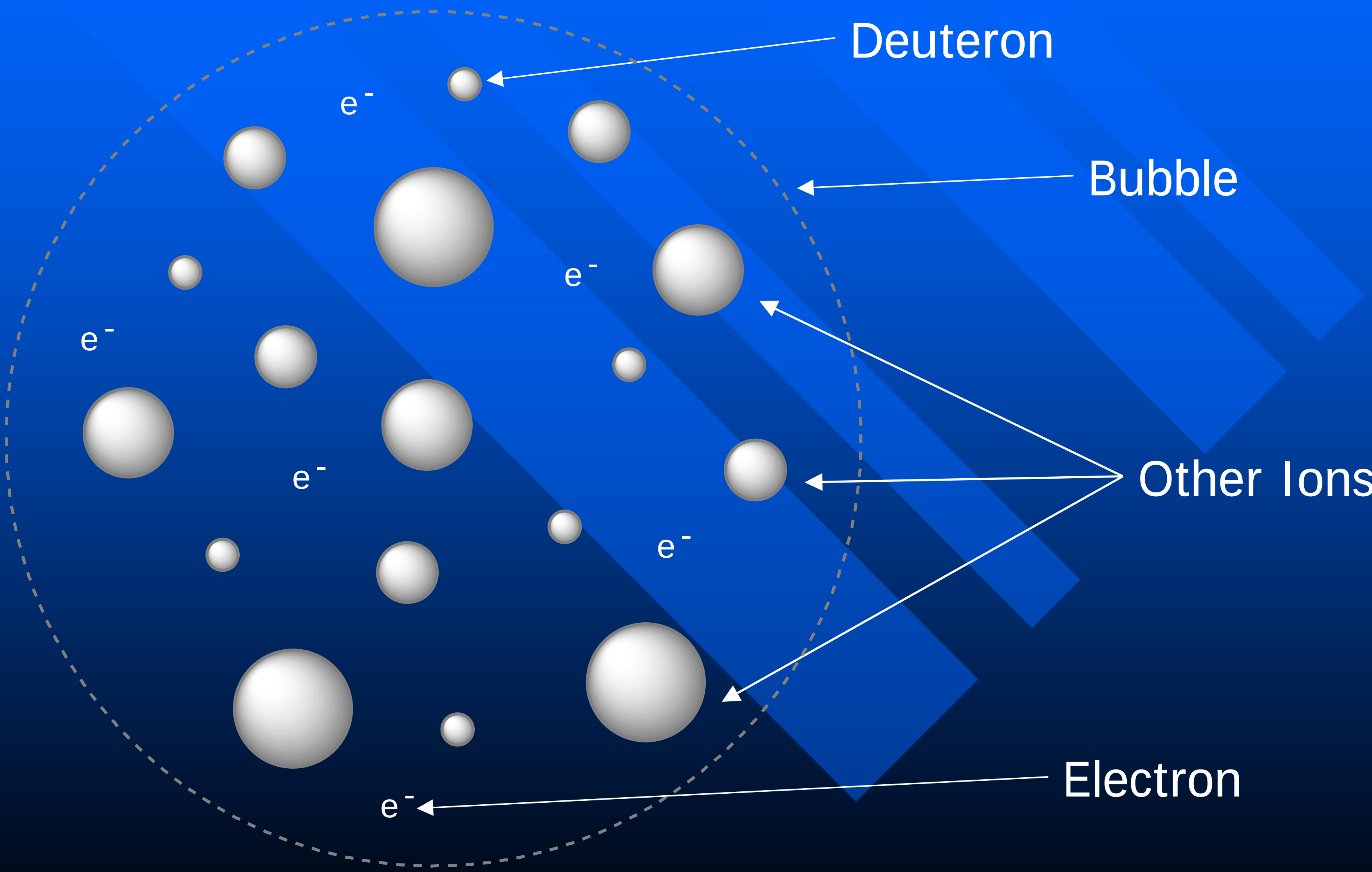
  Tritium production rate of ~$7x10^5$/sec. and neutron rate of ~$5x10^5$/sec.
  Neutron production is controversial (Shapira and Saltmarsh, Phys. Rev. Lett. **89**, 104302-1, (2002))

II.3.a Theoretical Model

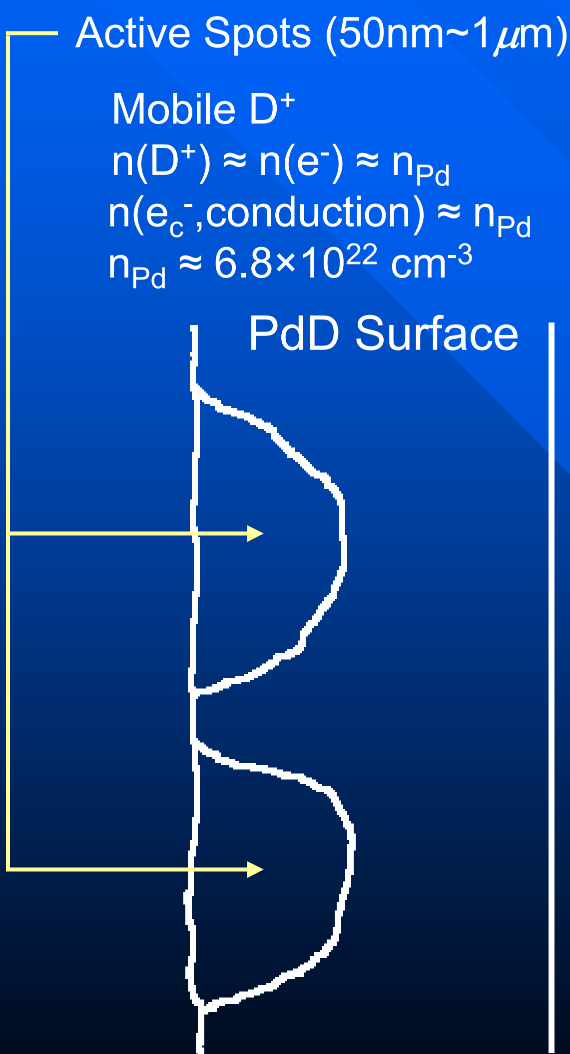
10 nm – 100 nm Atomic Cluster



Acoustic Cavitation Bubble (~10 nm – 1 mm)

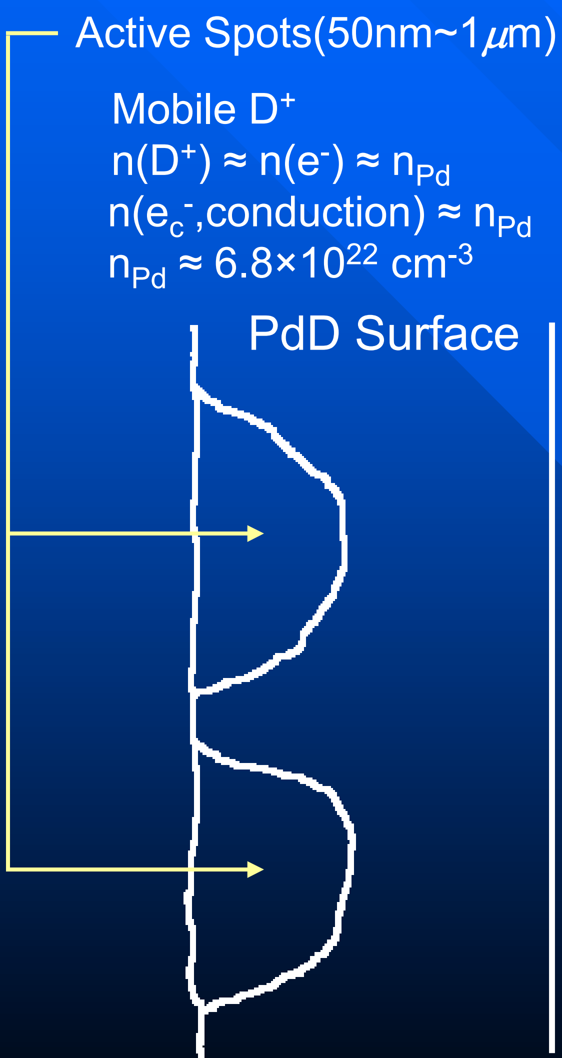


Proposed BEC Mechanism for LENR



- Electrolysis (Fleishmann & Pons, et. al.)
- Transient Cavitation Bubble (Stringham)
- Active sports are observed by Szpak, et. al., Dash, and others.
- Consistent with Arata-Zhang experiments
- Consistent with Ed Storms' "crud" scenario
- Consistent with Kasagi's LENR with low-energy deuteron beams

Proposed BEC Mechanism for LENR (continued)



- One Possible LENR channel

$$(N)D^+(BEC) \rightarrow (N\text{-}2)D^+(BEC \text{ or breakup}) + {}^4He$$
$$(D + D \rightarrow {}^4He)$$

- Temperature T of nD^+: Defined by the Bose-Einstein distribution

$$n(E) = \frac{1}{e^{\alpha}e^{E/kT} - 1} \rightarrow e^{-\alpha}e^{-E/kT}, \; E >> kT$$

- If $E_{D+} > E_{e_c-}$, conduction electrons can slow down D^+'s (electron cooling)
- At higher external temperatures, number of active spots will increase but T for D^+'s may not be affected to the same extent.
- Heat after death is possible if active spots have lifetimes.

Requirements for the BEC Mechanism for LENR (assumed)

- Non-equilibrium fluctuations will be able to make vacancies or impurities sufficiently mobile so as to allow Bose nuclei to move collectively in localized regions.
- Average de Broglie wavelength of Bose nuclei is comparable to or greater than average distance between Bose nuclei, thus providing "Quantum Coherence".
- Sufficiently large number of mobile Bose nuclei is present in a localized region.

Bose-Einstein Condensate Ground State

$$\Psi_{g.s.} = a\Psi_n(normal\ component) + b\Psi(condensate)$$

$$\langle \Psi_{g.s.} | \Psi_{g.s.} \rangle = 1 \rightarrow a^2 + b^2 = 1$$

$$a^2 \approx 1$$

$$b^2 = \Omega \approx 10^{-10} \sim 10^{-20}\ for\ LENR\ at\ ambient\ temperature$$

$$\Omega \approx 1 \quad for\ atomic\ BEC\ at\ T \approx n\ {}^oK$$

N-Body Schroedinger Equation

For both sonoluminescence and induced fusion, we consider N identical charged Bose nuclei confined in an ion trap (atomic cluster or bubble). For simplicity, we assume an isotropic harmonic potential for the ion trap in order to obtain order of magnitude estimates of fusion reaction rates.

N-body Schroedinger equation for the system is

$$\boxed{H\Psi = E\Psi} \qquad (1)$$

where Hamiltonian is given by

$$H = \frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \frac{1}{2}m\omega^2\sum_{i=1}^{N}r_i^2 + \sum_{i<j}\frac{e^2}{|r_i - r_j|} \qquad (2)$$

where m is the rest mass of the nucleus. In presence of electrons, we use shielded Coulomb potential.

II.3.b Equivalent Linear Two-Body (ELTB) Method

(Kim and Zubarev, Physical Review A **66**, 053602 (2002))

For the ground-state wave function Ψ, we use the following approximation

$$\Psi(\vec{r},\ldots\vec{r}_N) \approx \tilde{\Psi}(\rho) = \frac{\Phi(\rho)}{\rho^{(3N-1)/2}} \tag{3}$$

where $\rho = \left[\sum_{i=1}^{N} r_i^2\right]^{1/2}$

It has been shown that approximation (3) yields good results for the case of large N (Kim and Zubarev, J. Phys. B: At. Mol. Opt. Phys. **33**, 55 (2000))

By requiring that $\tilde{\Psi}$ must satisfy a variational principle $\delta\int\tilde{\Psi}^* H\tilde{\Psi}d\tau = 0$ with a subsidiary condition $\int\tilde{\Psi}^*\tilde{\Psi}d\tau = 1$, we obtain the following Schrödinger equation for the ground state wave function Φ(ρ)

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + \frac{m}{2}\omega^2\rho^2 + \frac{\hbar^2}{2m}\frac{(3N-1)(3N-3)}{4\rho^2} + V(\rho)\right]\Phi = E\Phi \tag{4}$$

where $$V(\rho) = \frac{2N\Gamma(3N/2)}{3\sqrt{2\pi}\Gamma(3N/2-3/2)\rho} \tag{5}$$

II.3.c Optical Theorem Formulation of Nuclear Fusion Reactions (Kim and Zubarev, Few-Body Systems Supplement 8, 324 (1995))

In order to parameterize the short-range nuclear force, we use the optical theorem formulation of nuclear fusion reactions. The total elastic nucleus-nucleus amplitude can be written as

$$f(\theta) = f^{c}(\theta) + \tilde{f}(\theta) \qquad (6)$$

where $f^{c}(\theta)$ is the Coulomb amplitude, and $\tilde{f}(\theta)$ can be expanded in partial waves

$$\tilde{f}(\theta) = \sum_{l} (2l+1) e^{2i\delta_l^c} f_l^{n(el)} P_l(\cos\theta) \qquad (7)$$

In Eq. (7), δ_l^c is the Coulomb phase shift, $f_l^{n(el)} = (S_l^n - 1)/2ik$, and s_l^n is the *l*-th partial wave S-matrix for the nuclear part.

For low energy, we can write (optical theorem)

$$\operatorname{Im} f_l^{n(el)} \approx \frac{k}{4\pi}\sigma_l^r \qquad (8)$$

where σ_l^r is the partial wave reaction cross section.

In terms of the partial wave t-matrix, the elastic scattering amplitude, $f_l^{n(el)}$ can be written as

$$f_l^{n(el)} = -\frac{2\mu}{\hbar^2 k^2} < \psi_l^c | t_l | \psi_l^c > \qquad (9)$$

where ψ_l^c is the Coulomb wave function.

II.3.d Parameterization of the Short-Range Nuclear Force

For the dominant contribution of only *s*-wave, we have

$$\mathrm{Im}\ f_0^{n(el)} \approx \frac{k}{4\pi}\sigma^r \quad (10)$$

and

$$f_0^{n(el)} = -\frac{2\mu}{\hbar^2 k^2}\langle \psi_0^c | t_0 | \psi_0^c \rangle \quad (11)$$

Where σ^r is conventionally parameterized as

$$\sigma^r = \frac{S}{E} e^{-2\pi\eta} \quad (12)$$

$\eta = \frac{1}{2kr_B}, r_B = \frac{\hbar^2}{2\mu e^2}, \mu = m/2$, $e^{-2\pi\eta}$ is the "Gamow" factor,
and *S* is the *S*- factor for the nuclear fusion reaction between two nuclei.

From the above relations, Eqs. (10), (11), and (12), we have

$$\frac{k}{4\pi}\sigma^r = -\frac{2\mu}{\hbar^2 k^2} < \psi_0^c | \mathrm{Im}\ t_0 | \psi_0^c > \quad (13)$$

For the case of *N* Bose nuclei, to account for a short range nuclear force between two nuclei, we introduce the following Fermi pseudo-potential $V^F(\vec{r})$

$$\mathrm{Im}\ t_0 = \mathrm{Im}\ V^F(\vec{r}) = -\frac{A\hbar}{2}\delta(\vec{r}) \quad (14)$$

where the short-range nuclear-force constant *A* is determined from Eqs. (12) and (13) to be $A = 2Sr_B / \pi\hbar$.

For deuteron-deuteron (DD) fusion via reactions *D(d,p)T* and *D(d,n)*^{3}*He*,
the *S*-factor is *S* = 110 *KeV-barn.*

II.3.e Derivation of Fusion Probability and Rates

For N identical Bose nuclei confined in an ion trap, the nucleus-nucleus fusion rate is determined from the trapped ground state wave function ψ as

$$R_t = -\frac{2}{\hbar}\frac{\Sigma_{i<j} < \psi \left| \mathrm{Im}\ t_{ij} \right| \psi >}{< \psi | \psi >} \tag{15}$$

where $\mathrm{Im}\ t_{ij}$ is given by the Fermi potential Eq. (14), $\mathrm{Im}\ t_{ij} = -A\hbar\delta(\vec{r})/2$.

From Eq. (15), we obtain for a single trap

$$R_t = \sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)N n_B \tag{16}$$

where Ω is the probability of the ground state occupation, $\alpha = e^2/\hbar c, n_B = N/<r>^3$ is Bose nuclei density in a trap, and $B = 3Am/8\pi c$ with $A = 2Sr_B/\pi\hbar$

For the case of multiple ion traps (atomic clusters or bubbles), the total ion-trap nuclear fusion rate *R* per unit time and per unit volume, can be written as

$$R = n_t\sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)Nn_B \tag{17}$$

where n_t is a trap number density (number of traps per unit volume) and *N* is the average number of Bose nuclei in a trap.

II.3.f Theoretical Predictions

Our final theoretical formula for the total nuclear fusion rate *R* per unit time per unit volume is

$$R = n_t \sqrt{\frac{3}{4\pi}} \Omega B \alpha \left(\frac{\hbar c}{m} \right) N n_B \qquad (17)$$

where only one unknown parameter is the probability of the BEC ground-state, Ω.

Prediction 1: R does not depend on the Gamow factor in contrast to the conventional theory for nuclear fusion in free space. This is consistent with Dirac's conjecture ("The Principles of Quantum Mechanics" (second edition), Oxford 1935, Chapter IX, Section 62).

Prediction 2: R increases as the temperature decreases since Ω increases as the temperature decreases.

Prediction 3: R is proportional to $n_t N n_B = n_t N^2 \langle r \rangle^{-3}$ where N is the average number of Bose nuclei in a single trap (atomic cluster or bubble). $\langle r \rangle$ is the average size of a trap.

Implications

Prediction 1: Nuclear fusion rate R does not depend on the Gamow factor in contrast to the conventional theory for nuclear fusion in free space.

This could provide explanations for some of the claimed anomalous effects for low-energy nuclear reactions:

- Solid State D-D Fusion experiments with nano-scale atomic clusters
- Acoustic Cavitation Fusion ("bubble fusion") can occur at lower temperatures.

Prediction 2: Nuclear reaction rate R increases as the temperature decreases, since the BEC ground-state occupation probability Ω increases as the temperature decreases.

- Experimental test by measuring the temperature dependence at Liquid Nitrogen and Helium temperatures.

Huizenga's Three Miracles

- The BEC mechanism may be able to achieve two (1 and 2) of Huizenga's miracles:
  - 1. Overcome the Coulomb barrier
  - 2. Radiationless fusion
    - » $(N)D^{+}(BEC) \rightarrow (N\text{-}2)D^{+}$ (BEC or break-up) + ^{4}He
- The third miracle:
  - 3. Branching ratio $R = \frac{\sigma[D(d,p)T]}{\sigma[D(d,n)^{3}\mathrm{He}]} > 1$

  may be irrelevant, if tritium production is not from D(d,p)T reaction