Quantum Many-Body Theory and Bose-Einstein Condensation (BEC) Mechanism for Low Energy Nuclear Reactions (LENR) and Transmutation Process in Condensed Matters

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I. **Conventional Nuclear Reaction Theory-Two Body Nuclear Reaction in Free Space**

II. **Quantum Many Body Theory for Bose-Einstein Condensate State and Application to LENR**

III. **Proposal for Experimental Tests of BEC Mechanism**

IV. **Generalization to Low Energy Nuclear Transmutation**

V. **Concluding Remarks/Discussion**

History of Cold Fusion

1925 – J. Tandberg builds first Pd electrolysis device

1926 – F. Paneth & K. Peters report detection of He after passing H_2 over Pd, claim H's fuse into He. (Retract after discovering He contamination from glass, asbestos used in experiment)

1932 – Discovery of neutron (by J. Chadwick) and deuterium

1932 – Tandberg (w/T. Wilner) repeats experiment w/D instead of H to search for enhanced Paneth-Peters effect

1934 – Discovery of D(D,p)T and D(D, n)^{3}He reactions by Oliphant, Harteck, and Lord Rutherford

1934 – D(D,p)T experiment using p and T ionization tracks in a cloud chamber by Dee

1940's – Tandberg & Wilner bombard Pd sheets w/high energy D's, finally observe $D + D \rightarrow {}^3He + n$

1947 – F.C. Frank proposes muon-catalyzed fusion hypothesis:

$$\frac{m_\mu}{m_e} \approx 200 \approx \frac{a_0^e}{a_0^\mu}$$

→ replace an e in H_2 (or D_2) w/a μ draws the atoms 200 times closer, reducing the effective thickness of the Coulomb barrier from ~ a_0 to ~ $a_0/200$

History cont'd

1940's-50's – further development by A. Sakharov et al

1957 – L. Alvarez & E. Teller discover (by accident) muon-catalyzed fusion in bubble chamber experiments (μ $T_{1/2}$ too short to catalyze enough fusions to be useful energy source)

1977 – Soviets at Dubna predict resonance effect in D-T admixture that would enhance # of fusions

1982 – S. E. Jones begins research w/Los Alamos μ-beam to find resonance-enhanced muon-catalyzed fusion; finds effect, but it is still too small to produce energy breakeven

1986 – P. Palmer hypothesizes that anomalous ^{3}He/ ^{4}He ratio in volcanic emissions is due to D + D → ^{3}He + n; Jones builds electrolysis device to mimic and test this possible effect.

1984 – S. Pons and M. Fleischmann search for fusion in D-saturated Pd electrodes (electrolysis increases D saturation)

1989 – After Pons and Fleischmann announcement: A flood of "confirmation" devices and variations

I. Conventional Nuclear Reaction Theory-Two Body Nuclear Reaction in Free Space

1. Semi-Classical Approach-Gamow Factor - Non-Resonance Reaction
2. Reaction Barrier Transparency (RBT) – Resonance Reaction
3. Quantum-Mechanical Derivation - Optical Theorem Formulation

Possible Fusion Reactions Involved:

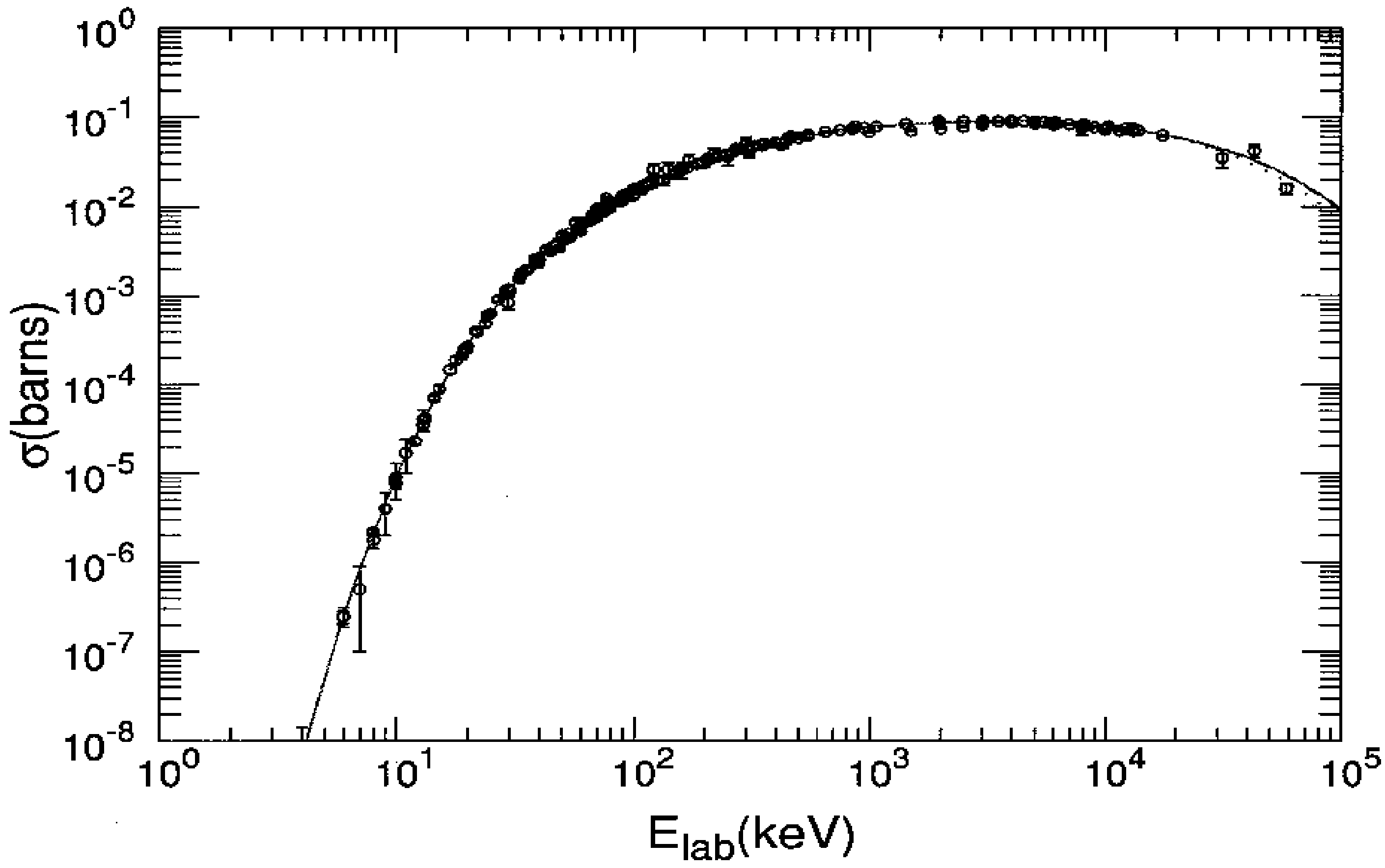
$$D + D \rightarrow {}^{3}H(1.01MeV) + p(3.02MeV)$$

$$D + D \rightarrow {}^{3}He(0.82MeV) + n(2.45MeV)$$

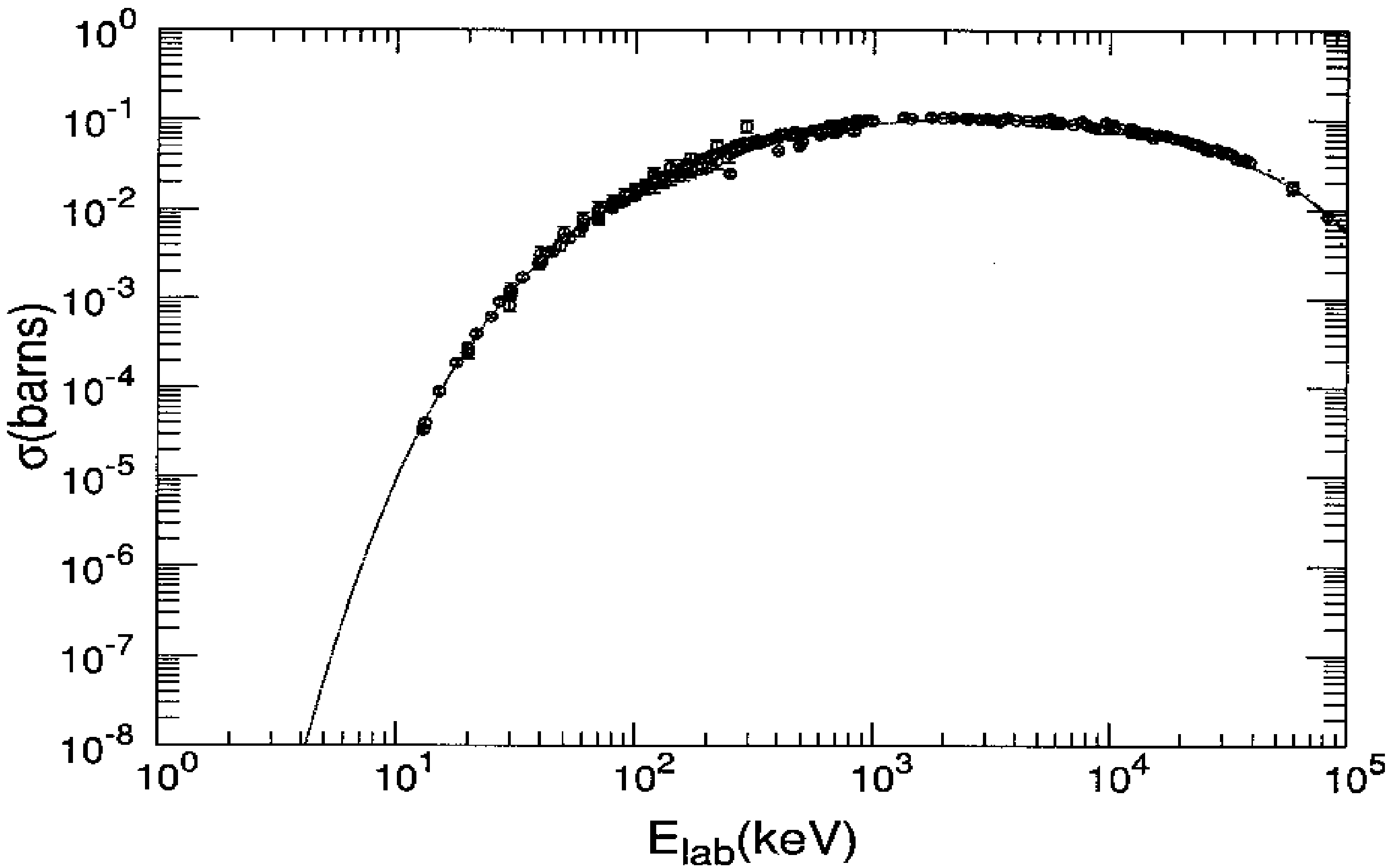
$$p + D \rightarrow {}^{3}He + \gamma(5.5MeV)$$

$$D + D \rightarrow {}^{4}He + \gamma(23.84MeV)$$

Cross-section for D(D,p)T



Cross-Section for D(D,n) ³He



Approximation for σ(E)

$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E),$$

$$\sigma_\ell(E) = \frac{\tilde{S}(E)}{E} T_\ell(E)$$

If the lowest partial wave ($\ell = 0$) contribution is expected to be dominant for low energies (< 20 keV), then

$$\sigma(E) \approx \sigma_0(E) = \frac{\tilde{S}(E)}{E} T_0(E)$$

<u>Models for $T_0(E)$</u>

Gamow Models: Gamow(1928), Fowler et al. (1967)

Kim-Zubarev Model: Mod. Phys. Lett. B7, 1627(1993); Int'l Theor. Phys. 33, 1889(1994); Transact. Fusion Tech. 226, 408(1994)

I. 1. Semi Classical Approach – Non-Resonance Reaction

Gamow Models

Wentzel-Kramers-Brillouin (WKB) approximation for the transmission coefficient

$$T_R^{WKB}(E)=\exp\left\{-2\sqrt{\frac{2\mu}{\hbar^2}}\int_R^{r_a}\left(\frac{Z_1Z_2e^2}{r}-E\right)^{1/2}dr\right\}$$

$$T_R^{WKB}(E)=\exp\left\{-\sqrt{\frac{E_G}{E}}\left(\frac{2}{\pi}\right)\left[\cos^{-1}\sqrt{\frac{E}{B}}-\sqrt{\frac{E}{B}}\sqrt{1-\frac{E}{B}}\right]\right\}$$

where $B=\frac{Z_1Z_2e^2}{R}, E=\frac{Z_1Z_2e^2}{r_a}$

For R=0 (or equivalently E<<B),

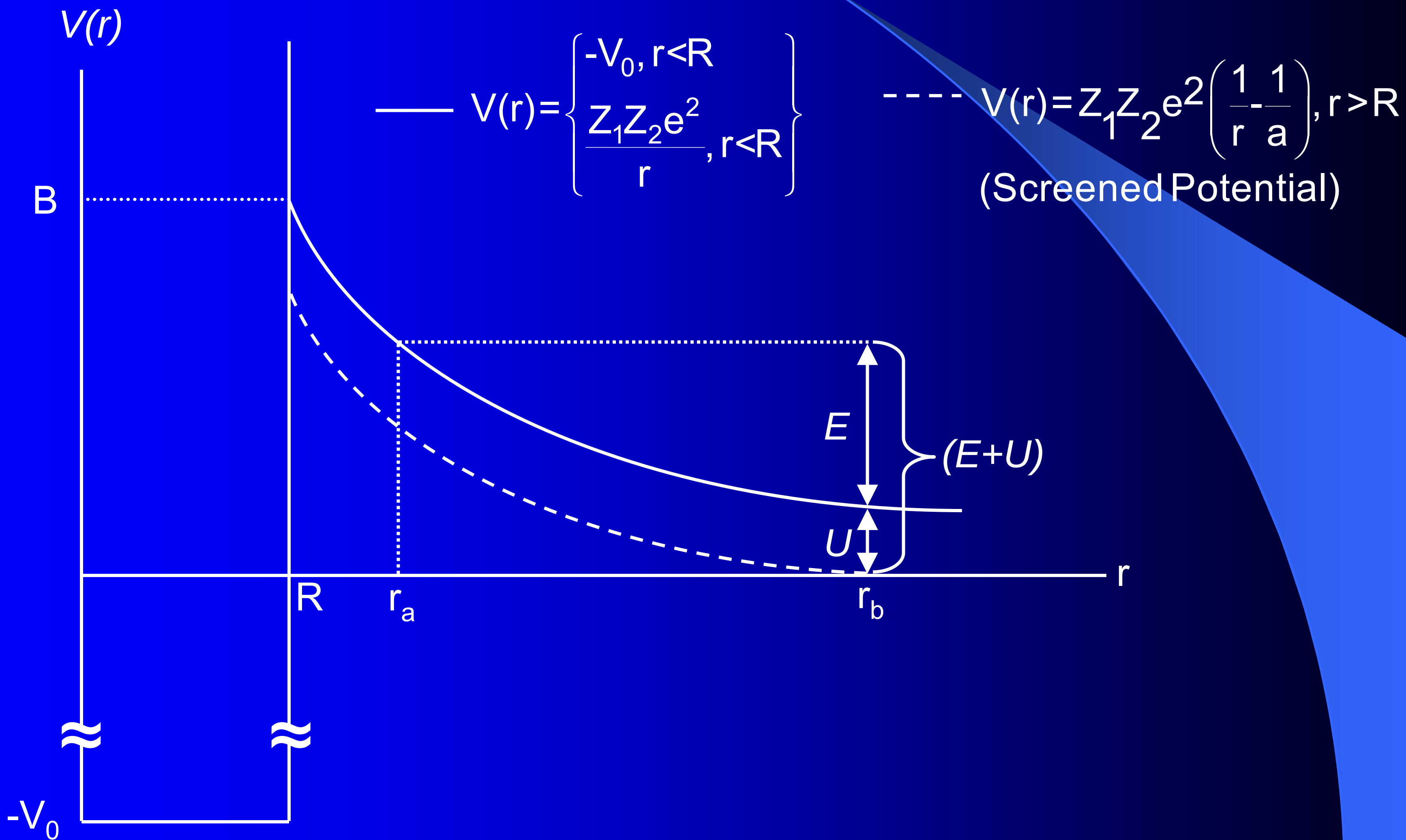
$$T_G(E)=T_{R=0}^{WKB}(E)=\exp\left\{-2\sqrt{\frac{2\mu}{\hbar^2}}\int_0^{r_a}\sqrt{\frac{Z_1Z_2e^2}{r}-E}\,dr\right\}$$

$$T_G(E)=\exp\left\{-\sqrt{\frac{E_G}{E}}\right\}$$

$$T_G(E)=e^{-\sqrt{E_G/E}} \quad (\text{"Gamow" Factor})$$

$$E_G=\frac{(2\pi\alpha Z_1Z_2)^2\mu c^2}{2}$$

Coulomb potential and nuclear square well potential



- Estimates of the Gamow factor for D + D fusion with electron screening energy

$$T_G(E) = e^{-\sqrt{E_G}/\sqrt{E}} = e^{-2\pi\eta}, \quad \eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

$$E \to E + E_{screening}$$

$E+E_{screen}$	exp[-2πη]	$E_{screening}$	$r_{screening}$
1/40 eV	10^{-2760}	0	
14.4 EV	10^{-114}	14.4 eV	1 Å
43.4 eV	10^{-65}	43.4 eV	0.33 Å
~300 eV	10^{-25}	300 eV	
~600 eV	10^{-18}	600 eV	

•Extracted values of Gamow Factor exp[-2πη] extracted from experiments

$$\left(e^{-2\pi\eta}\right)^{FP} \approx 10^{-20} \quad \text{(Fleishmann and Pons)}$$

$$\left(e^{-2\pi\eta}\right)^{Jones} \approx 10^{-30} \quad \text{(Jones, et al.)}$$

Quantum Scattering Theory for Two-Body Reaction with Two Potentials
(Nuclear and Coulomb potentials, $V^s + V^c$)

- Entrance Channel → Exit Channels

$$a + X \to \begin{cases} X + a & \text{elastic channel, } \sigma^{(el)} \\ X^* + a & \text{inelastic channel, } \sigma^{(inel)} \\ Y + b & \text{reaction (fusion) channel, } \sigma^{(r)} \end{cases}$$

- $\sigma^{(total)} = \sigma^{(e\ell)} + \sigma^{(r)}$

$$\sigma^{(r)} = \sum_{\ell} (2\ell+1)\sigma_{\ell}^{(r)}, \quad \sigma_{\ell}^{(r)} = \frac{\pi}{k^2}\left(1 - |\eta_{\ell}|^2\right)$$

$$\sigma^{(e\ell)} = \sum_{\ell} (2\ell+1)\sigma_{\ell}^{e\ell}, \quad \sigma_{\ell}^{e\ell} = \left| f_{\ell}^{e\ell} \right|^2$$

$$f_{\ell}^{e\ell} = f_{\ell}^{C} + e^{2i\delta_{\ell}^{C}} f_{\ell}^{N(e\ell)}, \quad f_{\ell}^{N(e\ell)} = \frac{\eta_{\ell} - 1}{2ik}$$

$$\sigma_{\ell}^{e\ell} = \left| f_{\ell}^{e\ell} \right|^2 = \frac{\pi}{k^2}\left\{ 4\sin^2\delta_{\ell}^{C} - 2\,\mathrm{Re}\left[(\eta_{\ell} - 1)\left(e^{2i\delta_{\ell}^{C}} - 1 \right) \right] + |\eta_{\ell} - 1|^2 \right\}$$

$$\text{or } \sigma_{\ell}^{e\ell} = \sigma_{\ell}^{C}(Rutherford) + \sigma_{\ell}^{nterference} + \sigma_{\ell}^{N(e\ell)}$$

The partial wave S-matrix, η_{ℓ}, is determined from ($V^s + V^c$)

- Conventional Approximation:

$$T_\ell(E) \propto \left(1 - |\eta_\ell|^2\right)$$

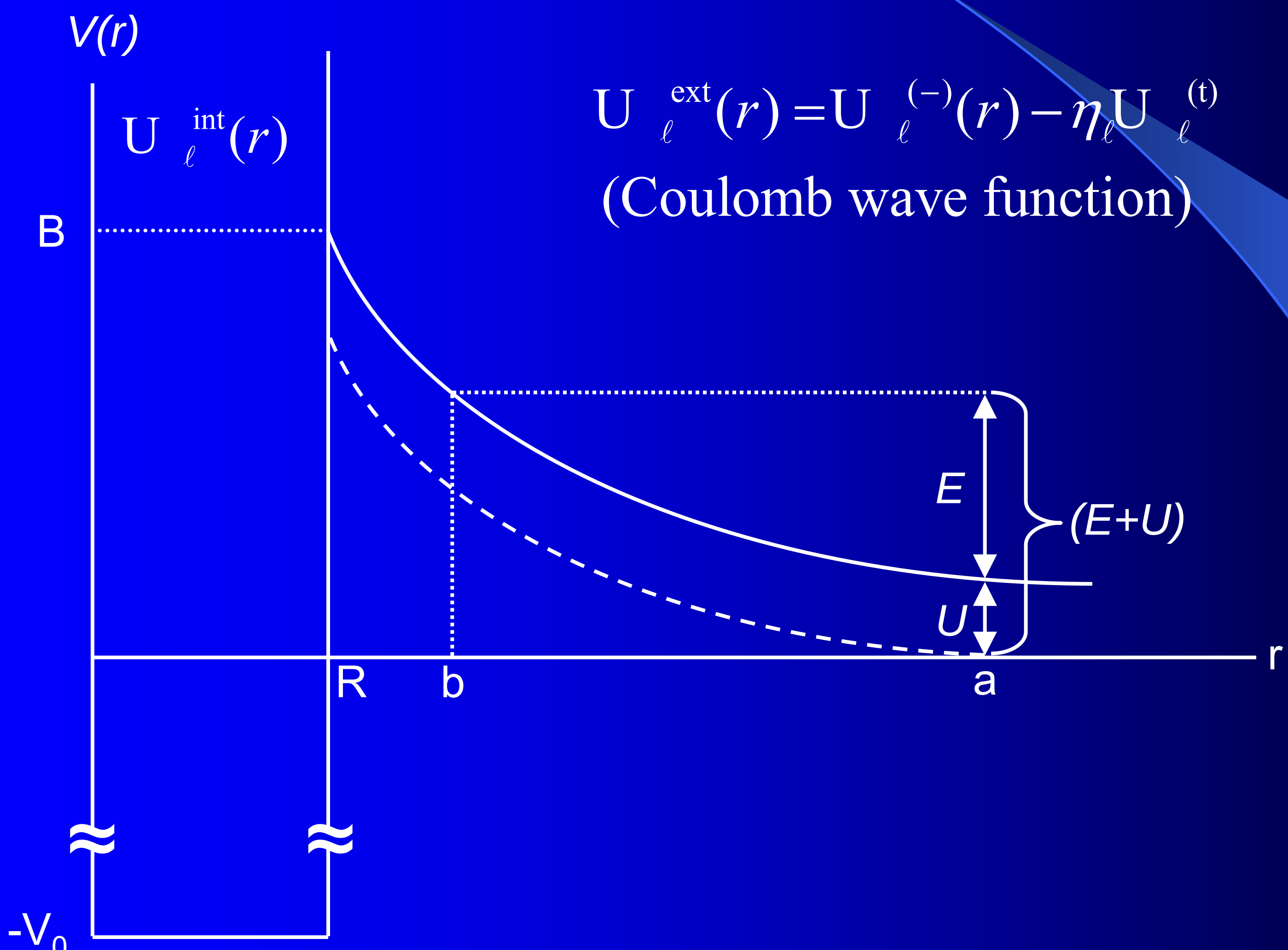
$$\sigma^{(r)}(E) = \frac{\pi}{k^2}\sum_{\ell=0}(2\ell+1)\left(1-|\eta_\ell|^2\right) \approx \frac{1}{E}\sum_{\ell=0}\tilde{S}^{\ell}(E)T_\ell(E)$$

where $S^{(\ell)}$ is the astronomical S-factor and $T_{(\ell)}(E)$ is the transmission coefficient (Gamow factor for the semi-classical approach)

Coulomb potential and nuclear square well potential

$$U_\ell^{ext}(r) = U_\ell^{(-)}(r) - \eta_\ell U_\ell^{(t)}$$

(Coulomb wave function)



I. 2. Reaction Barrier Transparency (RBT)- Resonance Reaction

General Formulae for S-Matrix (η_ℓ) and $T_\ell^{KZ} = 1 - |\eta_\ell|^2$

$$P_\ell^{int} = R \left. \frac{\dfrac{dU_\ell^{int}}{dr}}{U_\ell^{int}} \right|_{r=R} = Re P_\ell^{int} + i \operatorname{Im} P_\ell^{int} = \overline{K_2} - i\overline{K_1}$$

$$U_\ell^{ext} = U_\ell^{(-)} - \eta_\ell U_\ell^{(+)} \qquad P_\ell^{ext} = R \left. \frac{\dfrac{dU_\ell^{ext}}{dr}}{U_\ell^{ext}} \right|_{r=R} = R \frac{U_\ell^{(-)\prime} - \eta_\ell U_\ell^{(+)\prime}}{U_\ell^{(-)} - \eta_\ell U_\ell^{(+)}}$$

Require $\quad P_\ell^{int} = P_\ell^{ext} \qquad \eta_\ell = \left. \dfrac{P_\ell^{ext} U_\ell^{(-)} - R U_\ell^{(-)\prime}}{U_\ell^{(+)} \left[P_\ell^{int} - R \dfrac{U_\ell^{(+)\prime}}{U_\ell^{(+)}} \right]} \right|_{r=R}$

$$U_\ell^{(\pm)} = e^{\pm i\sigma} \left[G_\ell(r) \pm i F_\ell(r) \right]$$

$$R \left. \frac{U_\ell^{(+)\prime}}{U_\ell^{(+)}} \right|_{r=R} = R \frac{G_\ell'(r) + iF_\ell'(r)}{G_\ell + iF_\ell} = \Delta_\ell + iS_\ell$$

$$\Delta_\ell = R \left. \frac{G_\ell G_\ell' + F_\ell F_\ell'}{G_\ell^2 + F_\ell^2} \right|_{r=R}, \qquad S_\ell = R \left. \frac{G_\ell F_\ell' - G_\ell' F_\ell}{G_\ell^2 + F_\ell^2} \right|_{r=R} = \frac{kR}{G_\ell^2(R) + F_\ell^2(R)}$$

Transmission Coefficient (Kim-Zubarev Model)

Final Result for σ(E) (Kim-Zubarev Model)

$$T_\ell^{KZ} = 1 - |\eta_\ell|^2, \quad \sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \sum_{\ell=0}^{\infty} \frac{\tilde{S}_\ell(E)}{E} T_\ell^{KZ}(E)$$

$$T_\ell^{KZ}(E) = \frac{4 S_\ell \bar{K}_1^{(\ell)} R}{\left(\Delta_\ell - \bar{K}_2^{(\ell)} R\right)^2 + \left(S_\ell + \bar{K}_1^{(\ell)} R\right)^2}$$

- Note that, when $\Delta_\ell \cong \bar{K}_2^{(\ell)} R \ \& \ S_\ell \cong \bar{K}_1^{(\ell)} R,\ T_\ell^{KZ}(E)$ has a resonance behavior, $T_\ell^{KZ}(E) \approx 1$
- The resonance behavior of $T_\ell^{KZ}(E)$ is a Coulomb barrier transmission (CBT) resonance transparency (RT) due to an interplay of Coulomb barrier and nuclear reactions.
- It is to be distinguished from the conventional resonances such as neutron capture resonances which are primarily due to the nuclear reaction.
- In the conventional (Gamow) model, resonances are described using the Breit-Wigner formula in the *S*-factor.

Resonance Transparency (RT)

- Estimate of the resonance transparency width for S-wave (ℓ=0) case [Kim & Zubarev, ICCF-4 (1993)]

$$T_0(E) = \frac{4S_0\bar{K}_1R}{\left(\Delta_0 - \bar{K}_2R\right)^2 + \left(S_0 + \bar{K}_1R\right)^2} \quad , \quad \underline{S_0 > 0, \quad K_1R > 0.}$$

Conditions for RT: $\Delta_0 + iS_0 = i\bar{K}_1R + \bar{K}_2R$ or $\Delta_0 = \bar{K}_2R$ & $S_0 = \bar{K}_1R$

to be Breit-Wigner type $E = E_r - i\frac{\Gamma}{2}$

$$\left.\begin{aligned} \Delta_0(E) &\cong \Delta_0(E_r) - i\frac{\Gamma}{2}\left(\frac{\partial\Delta_0}{\partial E}\right)_{E=E_r} \\ S_0(E) &\cong S_0(E_r) - i\frac{\Gamma}{2}\left(\frac{\partial\Delta_0}{\partial E}\right)_{E=E_r} \end{aligned}\right\} \text{Taylor Expansion}$$

$$\Gamma = \frac{2\left[S_0(E_r) + \bar{K}_1R\right]}{\left(\frac{\partial\Delta_0}{\partial E}\right)_{E=E_r}} \approx 10^7 \text{ eV} \left[S_0(E_r) + \bar{K}_1R\right]$$

$$\therefore \Gamma \approx 10^7 \text{ eV } T_G(E_r), \text{ \& } T_{KZ}(E_r) = \frac{10^7 \text{ eV } T_G(E_r)}{\Gamma}$$

$$\rightarrow \quad T_{KZ} \approx 10^{17}\, T_G(E_r) \text{ for } \Gamma = 10^{-10} \text{ eV}$$

$$T_{KZ} \approx 10^{107}\, T_G(E_r) \text{ for } \Gamma = 10^{-100} \text{ eV}$$

Fusion Rate with Equilibrium Velocity Distribution

$$\langle \sigma \upsilon \rangle = \int \sigma(\upsilon)\, \upsilon\, f(\upsilon)\, d^3\upsilon$$

$$= \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E)\, E\, e^{-E/kT}\, dE$$

$$\sigma(E) = \frac{S(E)}{E} T_{KZ}(E) \approx \frac{S(0)}{E} T_{KZ}(E)$$

$$\langle \sigma \upsilon \rangle_{\text{new}} = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} S(0) \int T_{KZ}(E)\, e^{-E/kT}\, dE$$

$$= \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} S(0)\, T_{KZ}(E_r)\, e^{-E/kT} \int_{E_r - \frac{\Gamma}{2}}^{E_r + \frac{\Gamma}{2}} dE$$

$$= \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} S(0)\, e^{-E_r/kT}\, \Gamma, \quad \Gamma \approx e^{-\sqrt{E_G}/\sqrt{E}}$$

$$\langle \sigma \upsilon \rangle_{\text{old}} = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} S(0) \int_0^\infty e^{-\sqrt{E_G}/\sqrt{E}}\, e^{-E/kT}\, dE \approx \langle \sigma \upsilon \rangle_{\text{new}}$$

I. 3. Quantum-Mechanical Derivation – Optical Theorem Formulation

Optical Theorem Formulation for Two-Potential Scattering Problems

$$\sigma^{(el)} = \sum_{\ell} (2\ell+1)\sigma_{\ell}^{(el)}$$

$$f_{\ell}^{(el)} = f_{\ell}^{C} + \frac{e^{2i\delta_{\ell}^{C}}}{2ik}(S_{\ell}-1) = f_{\ell}^{C} + e^{2i\delta_{\ell}^{C}} f_{\ell}^{N(el)}$$

$$\begin{cases} \sigma_{\ell}^{(el)} = \left|f_{\ell}^{(el)}\right|^2 = \dfrac{\pi}{k^2}\left\{4\sin^2\delta_{\ell}^{C} - 2\,\mathrm{Re}\left[(S_{\ell}-1)(e^{2i\delta_{\ell}^{C}}-1)\right] + \left|S_{\ell}-1\right|^2\right\} \\ \text{or } \sigma_{\ell}^{(el)} = \sigma_{\ell}^{C}(\text{Rutherford}) + \sigma_{\ell}^{\text{interference}} + \sigma_{\ell}^{N(el)} \end{cases}$$

$$\left.\begin{array}{l} f_{\ell}^{N(el)} = \dfrac{S_{\ell}-1}{2ik}, \quad \sigma_{\ell}^{N(el)} = \left|f_{\ell}^{(el)}\right|^2 = \dfrac{\pi}{k^2}\left|S_{\ell}-1\right|^2 \\ \sigma^{(r)} = \sum_{\ell}(2\ell+1)\sigma_{\ell}^{(r)}, \quad \sigma_{\ell}^{(r)} = \dfrac{\pi}{k^2}\left(1-\left|S_{\ell}\right|^2\right) \end{array}\right\}$$

where S_{ℓ} is the S-matrix.

Optical Theorem Formulation (Continued)

$$\begin{cases} \sigma_\ell^{(r)} + \sigma_\ell^{N(el)} = \dfrac{2\pi}{k^2}(1 - \operatorname{Re} S_\ell) \\ \operatorname{Im} f_\ell^{N(el)} = \dfrac{1}{2i}\left[f_\ell^{N(el)} - f_\ell^{N(el)*} \right] = \dfrac{1}{2k}(1 - \operatorname{Re} S_\ell) \end{cases}$$

$$\therefore \operatorname{Im} f_\ell^{N(el)} = \frac{k}{4\pi}\left(\sigma_\ell^{(r)} + \sigma_\ell^{N(el)} \right) \quad \text{(Optical Theorem, rigorous result)}$$

For low energies, $(S_\ell - 1) \propto e^{-2\pi\eta}$

$$\begin{cases} f_\ell^{N(el)} = \dfrac{S_\ell - 1}{2ik} = -\dfrac{2\mu}{\hbar^2 k^2}\left\langle \varphi_\ell^C \middle| T_\ell \middle| \varphi_\ell^C \right\rangle \propto \dfrac{e^{-2\pi\eta}}{k} \\ \sigma_\ell^{N(el)} = \left| f_\ell^{N(el)} \right|^2 \propto \dfrac{e^{-4\pi\eta}}{k^2} \\ \sigma_\ell^{(r)} \propto \dfrac{e^{-2\pi\eta}}{k^2} \quad \& \quad \sigma_\ell^{(r)} >> \sigma_\ell^{N(el)} \quad \text{(at low energies)} \end{cases}$$

$$\operatorname{Im} f_\ell^{N(el)} \approx \frac{k}{4\pi}\sigma_\ell^{(r)} \quad \text{(Optical Theorem at low energies)}$$

Optical Theorem Formulation (continued)

Optical Theorem Formulation of $\sigma(E)$

$$\sigma(E) = \sigma^{(r)}(E) = \sum_{\ell}(2\ell+1)\sigma_{\ell}^{(r)},\ T = V^{S} + V^{S}\frac{1}{E - H_0 - V^{C}}T$$

$$\operatorname{Im} f_{\ell}^{N(el)} = \frac{k}{4\pi}\left(\sigma_{\ell}^{(r)} + _{\ell}^{N(el)}\right) \approx \frac{k}{4\pi}\sigma_{\ell}^{(r)} \quad \text{(for low energies)}$$

$$\therefore \sigma_{\ell}^{(r)}(E) = \frac{4\pi}{k}\operatorname{Im} f_{\ell}^{N(el)} = \frac{4\pi}{kE}\left\{-\operatorname{Im}\left\langle \varphi_{\ell}^{C}\middle| T_{\ell} \middle| \varphi_{\ell}^{C}\right\rangle\right\}$$

$$\sigma_{\ell}^{(r)}(E) = \frac{4\pi}{kE}\int_0^{\infty}\int_0^{\infty}\varphi_{\ell}^{C}(r)U_{\ell}(r,r')\varphi_{\ell}^{C}(r')drdr'$$

$$U_{\ell}(r,r') = -\operatorname{Im}\langle r|T_{\ell}|r'\rangle$$

Optical Theorem Formulation – Nuclear Interaction

For 2-channel (elastic and fusion) case with $\ell = 0$

$$U_0(r,r') = \lambda g(r) g(r')$$

$$\sigma_0^{(r)}(E) \equiv \sigma_0(E) = \frac{4\pi\lambda}{kE}\left[\int_0^\infty dr \varphi_0^C(r) g(r)\right]^2$$

$$= \frac{4\pi^2\lambda}{E} R_B \frac{(e^{-2\phi\eta}-1)^2}{(e^{2\phi\eta}-1)} e^{4\phi\eta} \text{ with } g(r) = \frac{e^{-\beta r}}{r}$$

$$\sigma_0^{(r)}(E) = \frac{S_0(E)}{E} e^{-2\hbar\eta} e^{4\phi\eta}$$

$$e^{4\phi\eta} = e^{4\eta\tan^{-1}(k/\beta)}, \quad \eta = \frac{Z_a Z_b e^2}{\hbar\upsilon} = \frac{1}{2kR_B}, \quad R_B = \frac{\hbar^2}{2\mu Z_a Z_b e^2},$$

$$S_0(E) = 4\pi^2\lambda R_B (e^{-2\phi\eta}-1)^2$$