

Basics of Deuteron-Cluster Dynamics by Langevin Equation

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To be presented at ACS2009

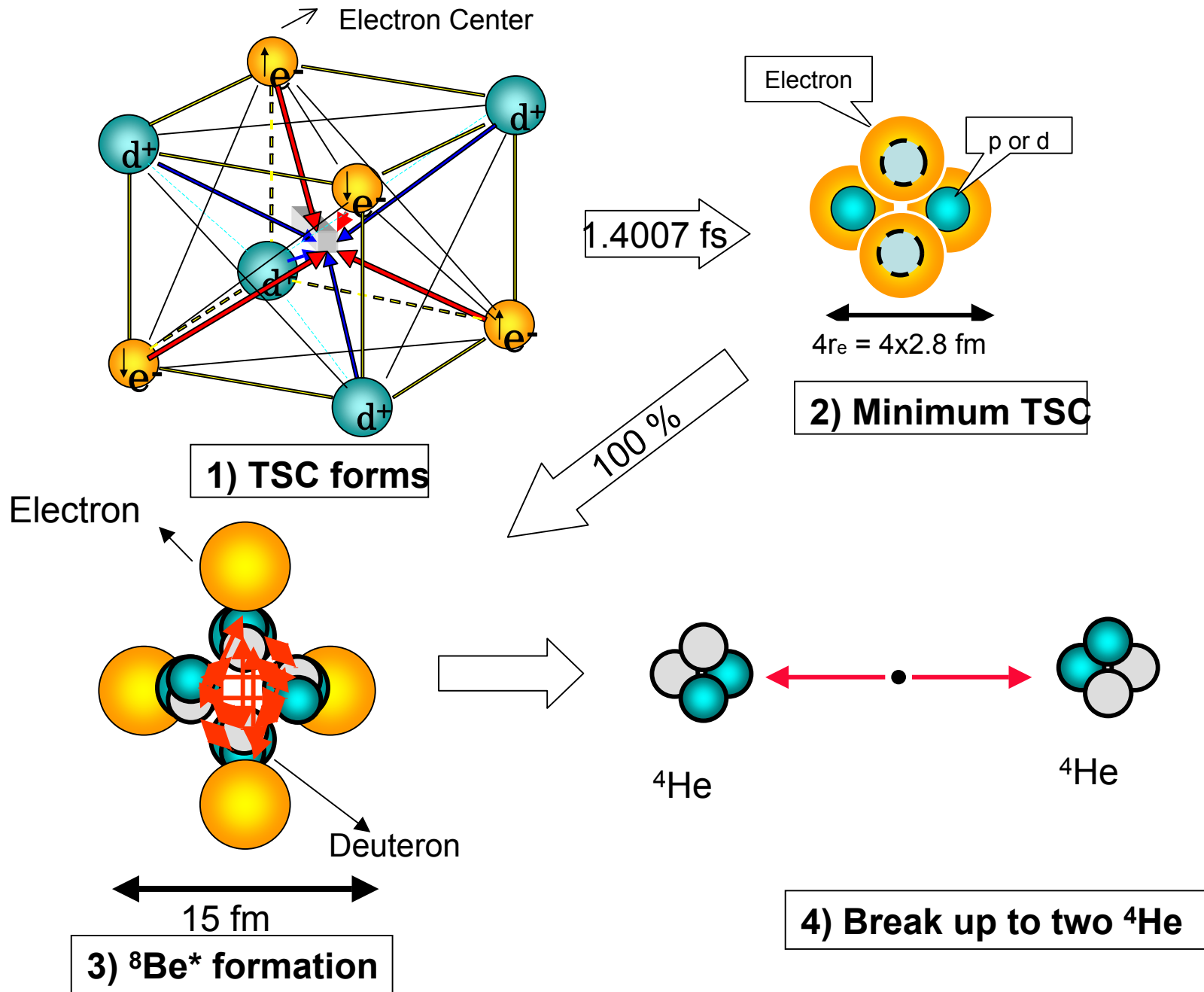
NET Symposium, Salt Lake City

March 2009

1. Outline

- Generation and condensation of 4D/TSC as seed of clean fusion in condensed matter was previously studied by Langevin equation
- Basics of Langevin Equation for D-cluster is treated in this study
- Known D-systems: D, D₂, D₂⁺, D₃⁺
- 4D/TSC and 6D/OSC
- Barrier factors and fusion rates

Result of Dynamic Condensation of 4D/TSC by Langevin Equation



Case-1

Model for generating 4D/TSC ($t=0$) cluster in excited state dynamics of O-site Deuterons in PdD lattice, where D behaves as a harmonic oscillator

A. Takahashi and N. Yabuuchi / Journal of Condensed Matter Nuclear Science 1 (2007) 1-23

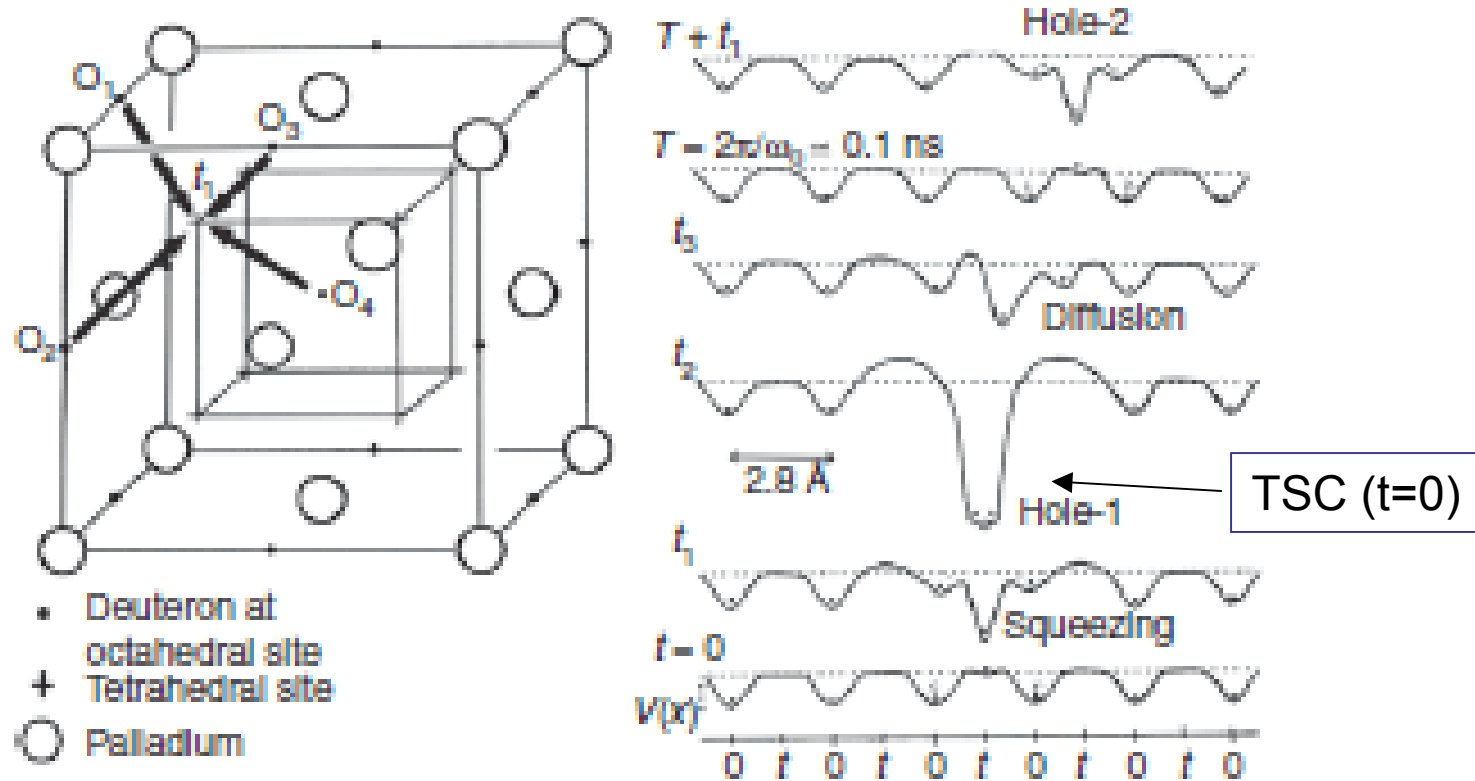


Figure 8. Image of lattice potential change by D-phonon excitation.

Under the stimulation of D-phonon excitation in PdD_x (suppose locally x=1) lattice, transient 4D-cluster will be formed with certain probability [28]. We know the trapping periodical potential height for D in PdD_x lattice is about 0.22 eV, and we use kinetic energy of deuteron $E_d = 0.22\text{eV}$ for numerical estimation in the following. To keep charge neutral state in average, orthogonal combination of two transient D₂ molecules will be formed when 4 deuterons are squeezing from O-sites to central T-sites taking (conveying) 4 electrons from Pd 4d-shell (conduction band), as shown in Fig.1, under the TSC condition and the cluster of 4D⁺s and 4e⁻s makes a regular cube as shown in Fig.2.

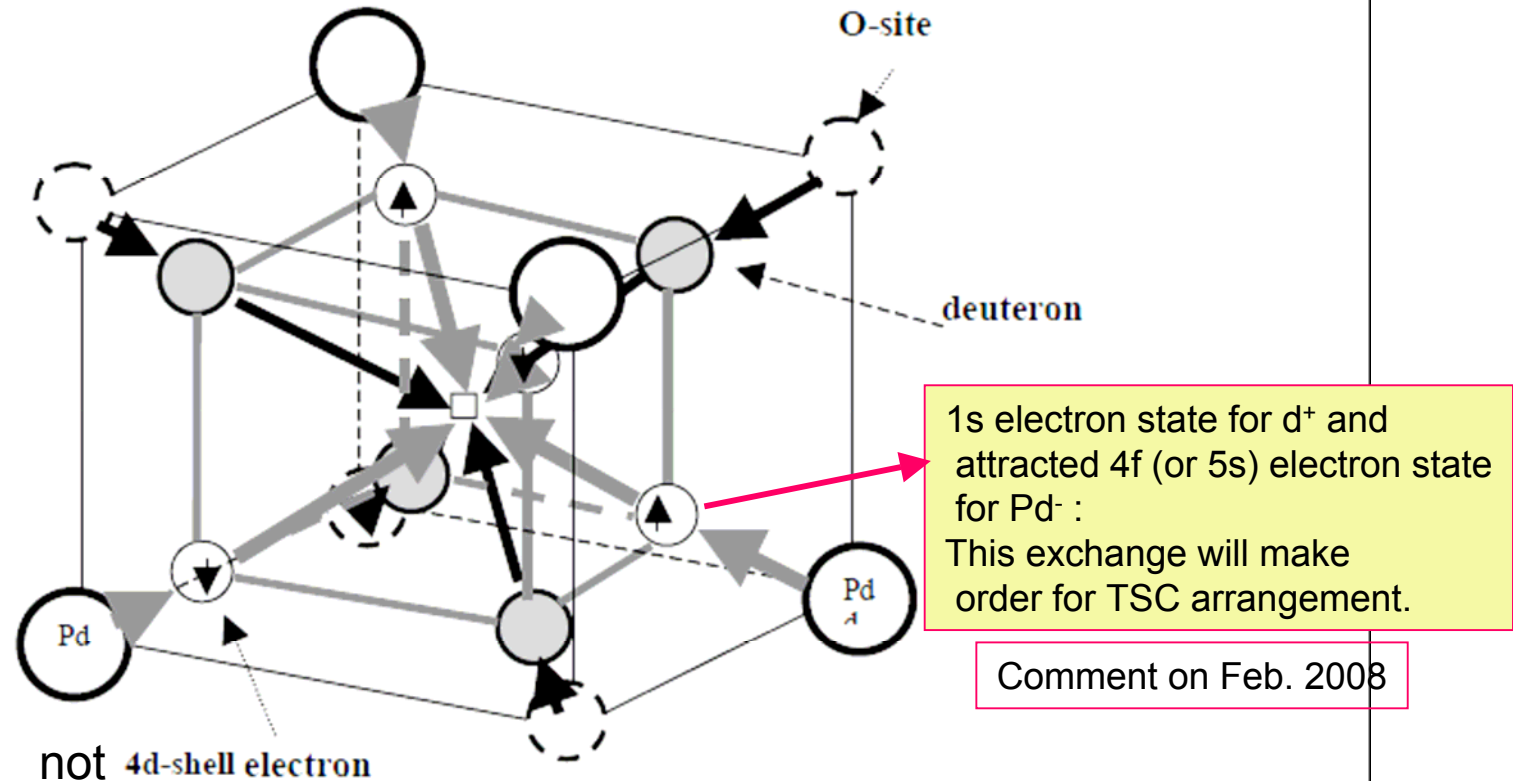
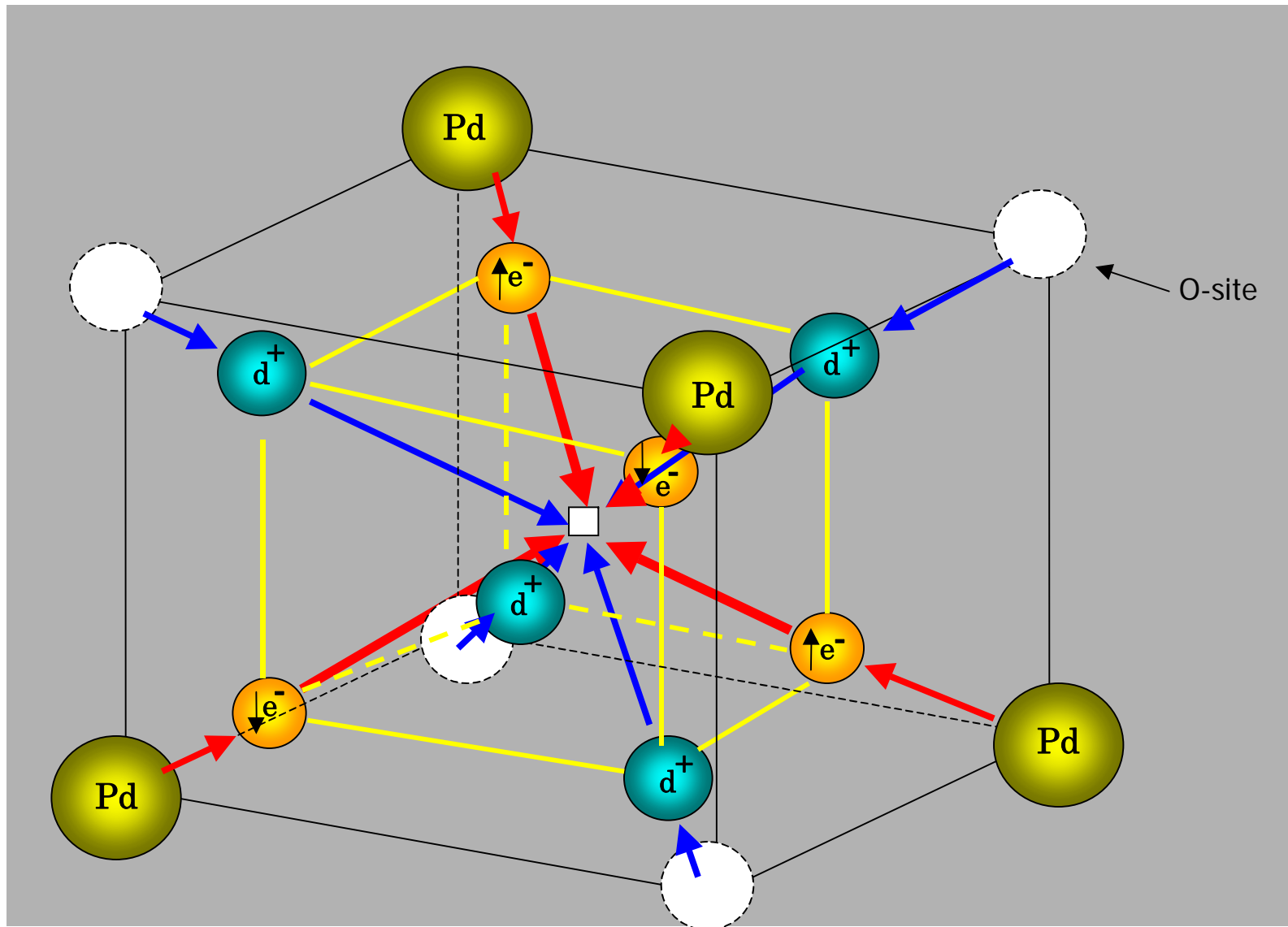


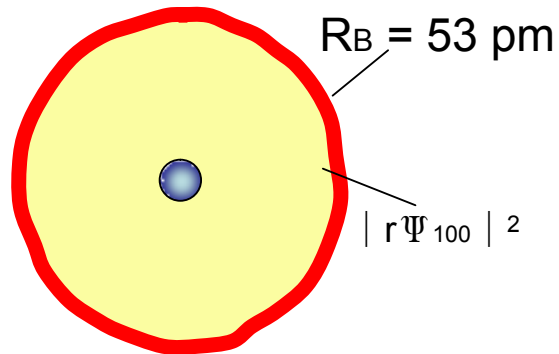
Figure 1. Tetrahedral condensation of deuterons in PdD lattice

Tetrahedral Condensation of Deuterons in PdDx

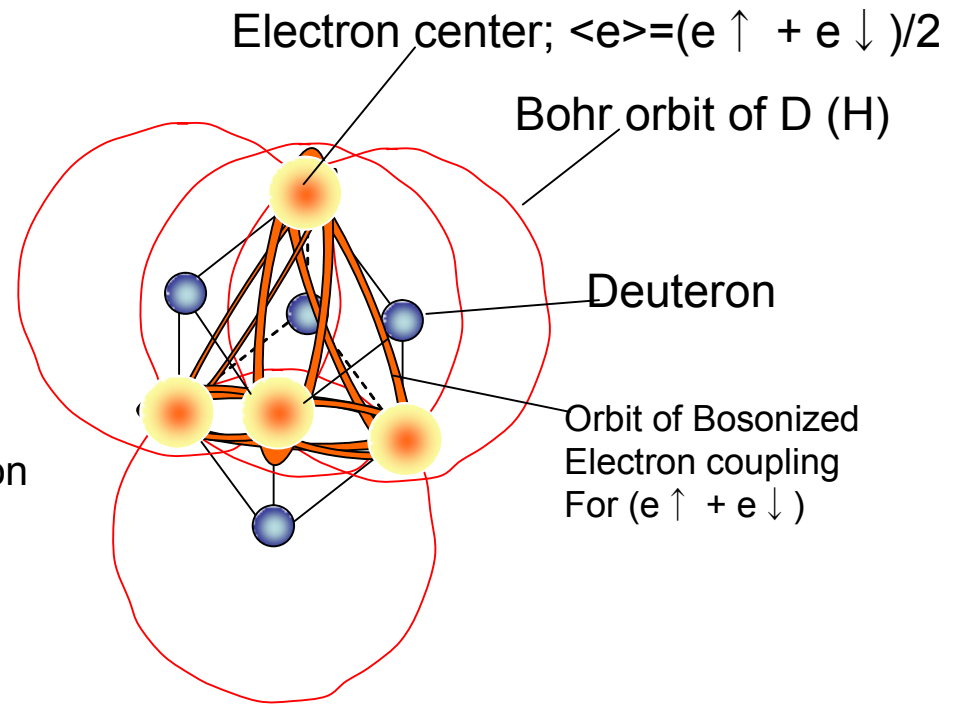


A. Takahashi: Proc. ICCF10, pp.809-818, World Scientific PC, 2006

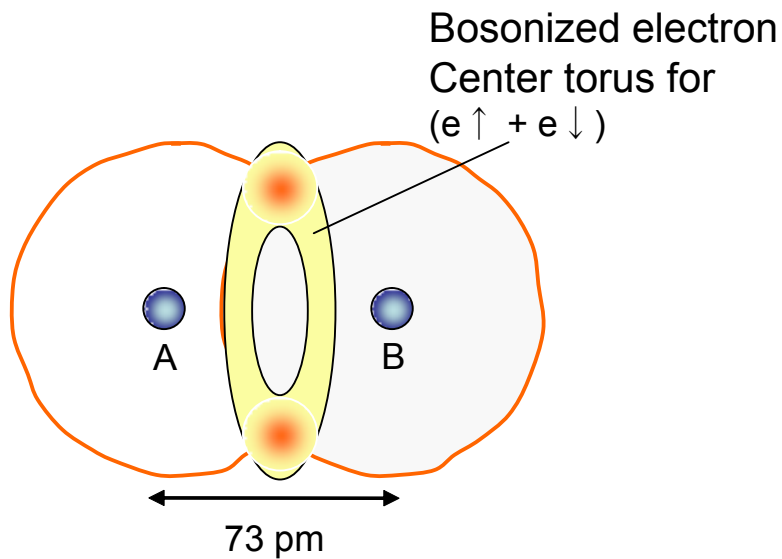
Feature of QM Electron Cloud



a) D atom (stable)



c) 4D/TSC (life time about 60 fs)



b) D₂ molecule (stable): $\Psi_{2D} = (2 + 2\Delta)^{-1/2} [\Psi_{100}(r_{A1}) \Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2}) \Psi_{100}(r_{B1})] X_s(S1, S2)$

Elaboration of 4D/TSC Models

- A. Takahashi: Deuteron cluster fusion and related nuclear reactions in metal-deuterium/hydrogen systems, *Recent. Res. Dev. Physics*, 6(2005)pp.1-28
- A. Takahashi: A theoretical summary of condensed matter nuclear effects, Proc. Siena05 Workshop, to be published in *JCMNS (2007)*
- A. Takahashi, N. Yabuuchi: Condensed matter nuclear effects under Platonic symmetry, Proc. ICCF13
- A. Takahashi, N. Yabuuchi: Study on 4D/TSC condensation motion by non-linear Langevin equation, Proc. New Energy Technologies, ACS, 2007 (published from Oxford U. Press, August 2008)
- And others in Proceedings of ICCF11-14.

This work: Milestones for solving the problem

- **So many body problem:** 4d +4x(D1s-electron)+ 4x10x(Pd 4d-shell electron)+4Pd + surrounding lattice atoms
- **Platonic Symmetry helps** the problem make simpler
- Formulate one-dimensional **Langevin equation for D-cluster dynamics**

One-Dimensional Langevin Equation for Molecule with Platonic Symmetry

- Formulate Langevin Equation with one-dimensional R_{dd} (d-d distance).
- Treat electron wave by combination of “dede” or “dde” type potentials.
- Solve Langevin Equation for D-clusters with Platonic symmetry for deuterons and electrons: D, D₂, D₂⁺, D₃⁺, 4D/TSC, 6D⁻/OSC.

2: D(H)-atom

- 1S-wave function

$$\Psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

System Coulomb Energy

$$\langle \Psi_{100} | E_{C-D} | \Psi_{100} \rangle = \int_0^\infty (-e^2 / r) \Psi_{100}^2 4\pi r^2 dr = -1.44 / r$$

With $r =$ Bohr radius (52.9 pm), we get

$$\langle E_{C-D} \rangle = -27.2 eV$$

D(H)-atom-II

- Total system energy is given by Hamiltonian integral:

$$\langle H \rangle = \langle \Psi_{100} | H | \Psi_{100} \rangle = \langle \Psi_{100} | -\frac{\hbar^2}{2m} \nabla^2 - \frac{e^2}{r} | \Psi_{100} \rangle = E_0 = -13.6 eV$$

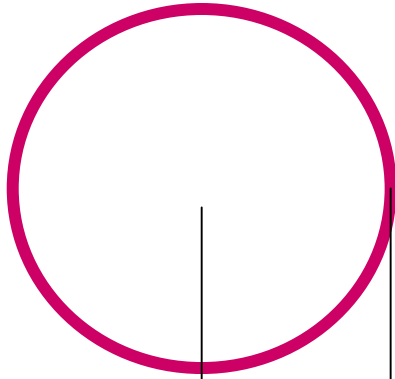
$$\langle H \rangle = \langle E_k \rangle + \langle E_{C-D} \rangle$$

Kinetic energy

Coulomb energy

$$\langle E_k \rangle = \frac{1}{2} m v^2 (r = a_B) = \frac{e^2}{2r} = 13.6 eV$$

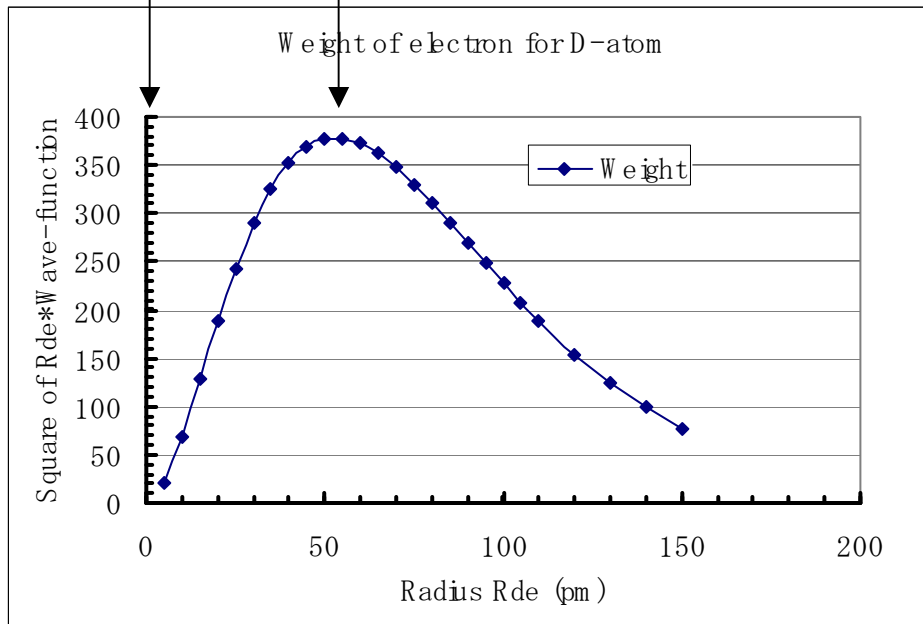
D-Atom: Point-Sphere Coupling



$$\Psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

$$\int_0^{\infty} (\Psi_{100}(r))^2 4\pi r^2 dr = \int_0^{\infty} 4\pi (r\Psi(r))^2 dr = 1$$

Peak at $R_{de}=a=R_B$
(52.9 pm)



Electron Weight Peak
Localizes at

$$R_{de} = a = R_B = 52.9 \text{ pm}$$

2.2) D-atom Langevin Equation

Regarding QM average = Ensemble Av.

$$m_e \frac{d^2 R_{de}}{dt^2} = -\frac{e^2}{[R_{de}]^2} + \frac{m_e v_e^2}{R_{de}} + f(t)$$

$$\langle G \rangle_{ensemble} = \langle \Psi | G | \Psi \rangle$$

$$m_e \frac{d^2 \langle R_{de} \rangle}{dt^2} = -\left\langle \frac{e^2}{R_{de}^2} \right\rangle + \left\langle \frac{m_e v_e^2}{R_{de}} \right\rangle = 0$$

$$\langle E_{KE} \rangle = \frac{1}{2} m_e \langle v_e^2 \rangle = \frac{e^2}{2R_B} = 13.6 eV$$

$$\langle E_C \rangle = -\frac{e^2}{R_B} = -27.2 eV$$

$$m_e \frac{d \langle R_{de} \rangle}{dt} = F(T) = \int_0^T f(t) dt = \langle f(t) \rangle = 0$$

$$\langle H \rangle = \langle E_{KE} \rangle + \langle E_C \rangle = -13.6 eV$$

Due to the ergodic process

$$\langle R_{de} \rangle(t) = R_0 = R_B = 52.9 pm$$

Balance of centripetal
And centrifugal force

Mean Electron Kinetic Energy = 13.6 eV ; 332 pm Wave Length = $2 \pi R_B$

3. D₂ molecule

- System wave function:

$$\Psi_{2D} = \frac{1}{\sqrt{(2 + 2\Delta)}} [\Psi_{100}(r_{A1})\Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2})\Psi_{100}(r_{B1})] X_s(S1, S2)$$

System Energy at ground state

$$\langle \Psi_{2D} | H | \Psi_{2D} \rangle = -35.1 eV$$

System Coulomb Energy: Semi-Classical model

$$\langle E_{C-2D} \rangle = 4\left(-\frac{e^2}{a_B}\right) + 2\left(\frac{e^2}{\sqrt{2}a_B}\right) = -70.3 eV$$

17.6 eV per e

Electron Kinetic E

$$\langle E_{ke-2D} \rangle = \langle H_{2D} \rangle - \langle E_{C-2D} \rangle = 70.3 eV - 35.1 eV = 35.2 eV$$

Quantum-Mechanical Ensemble-Averaging

$$\langle G \rangle_{ensemble} = \langle \Psi | G | \Psi \rangle$$

Born-Oppenheimer Approximation; for D₂ molecule:

$$\Psi(R_{dd}; r_{A1}, r_{A2}, r_{B1}, r_{B2}) = \Psi_{2D} \cdot X(R_{dd})$$

Electron Wave Function for D₂:

$$\Psi_{2D} = \frac{1}{\sqrt{(2 + 2\Delta)}} [\Psi_{100}(r_{A1})\Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2})\Psi_{100}(r_{B1})] X_s(S1, S2)$$

Deuteron Wave Function: Gaussian approximation:

$$X^2(R'_{dd}; R_{dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(R'_{dd} - R_{dd}(t))^2 / (2\sigma^2)]$$

D₂ Molecule Electron Localization; 1/2

$$\Psi_{2D} = \frac{1}{\sqrt{(2 + 2\Delta)}} [\Psi_{100}(r_{A1})\Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2})\Psi_{100}(r_{B1})] X_s(S1, S2)$$

$$(4\pi)^4 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \Psi_{2D}^2(r_{A1}, r_{A2}, r_{B1}, r_{B2}) r_{A1}^2 r_{A2}^2 r_{B1}^2 r_{B2}^2 dr_{A1} dr_{A2} dr_{B1} dr_{B2} = 1$$

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \rho(r_{A1}, r_{A2}, r_{B1}, r_{B2}) dr_{A1} dr_{A2} dr_{B1} dr_{B2} = 1$$

$$\rho(r_{A1}, r_{A2}, r_{B1}, r_{B2}) = \frac{(4\pi)^4}{2 + 2\Delta} \left\{ \begin{aligned} & [r_{A1}^2 \Psi_{100}^2(r_{A1}) r_{B2}^2 \Psi_{100}^2(r_{B2})] r_{A2}^2 r_{B1}^2 \\ & + 2[r_{A1} \Psi_{100}(r_{A1}) r_{A2} \Psi_{100}(r_{A2}) r_{B1} \Psi_{100}(r_{B1}) r_{B2} \Psi_{100}(r_{B2})] r_{A1} r_{A2} r_{B1} r_{B2} \\ & + [r_{A2}^2 \Psi_{100}^2(r_{A2}) r_{B1}^2 \Psi_{100}^2(r_{B1})] r_{A1}^2 r_{B2}^2 \end{aligned} \right\}$$

$(r\Psi)^2$ is a measure of electron weight localization

D₂ Molecule Electron Localization: 2/2

At $r_{A1} = r_{B1}$ and $r_{A2} = r_{B2}$:

$$\rho(r_{A1}, r_{A2}, r_{B1}, r_{B2}) = \frac{4(4\pi)^4}{2 + 2\Delta} \left[|r_{A1} \Psi_{100}(r_{A1})|^2 |r_{A2} \Psi_{100}(r_{A2})|^2 \right] r_{A1}^2 r_{A2}^2$$

Platonic System:
“Di-Cone”
Dipole-Circle Coupling

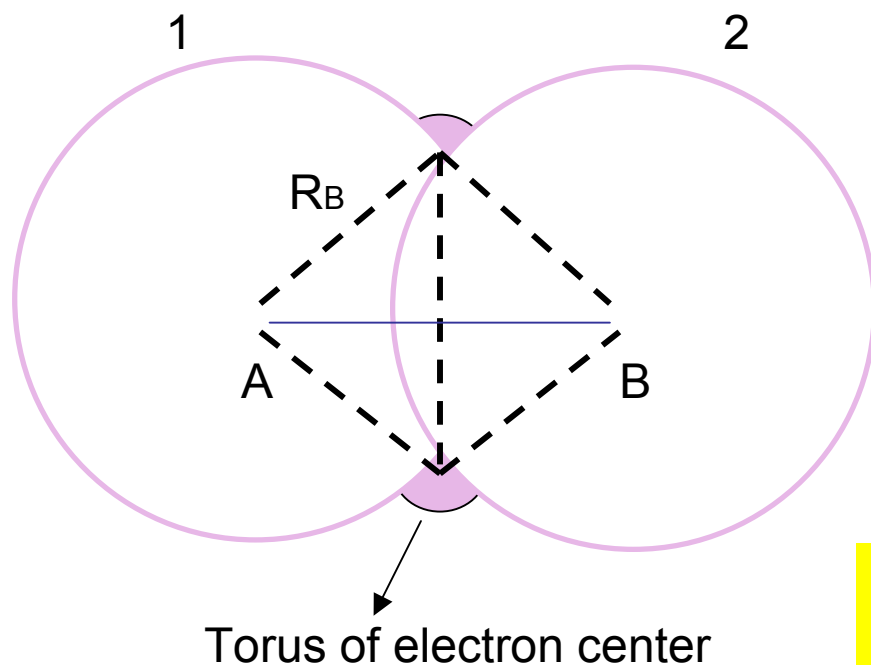
At $r_{A1} = r_{A2} = r_{B1} = r_{B2} = a$
Maximum appears.

$$\rho(a, a, a, a) = \frac{4(4\pi)^4}{2 + 2\Delta} a^4 |a \Psi_{100}(a)|^4$$

$$R_B = a = 52.9 \text{ pm}$$

$$R_{dd} = R_{AB} = 74.1 \text{ pm}$$

Classical view:
Electrons rotate around
 R_{dd} axis.



Mean Electron Kinetic Energy = 17.6 eV
; Wave length = 223 pm = $2 \pi R_e$

3.2) D₂ Langevin Equation

$$m_d \frac{d^2 R_{dd}}{dt^2} = \underbrace{-(4\sqrt{2} - 2) \frac{e^2}{R_{dd}^2}}_{\text{Coulomb F}} + \underbrace{\frac{2m_e v_e^2}{(R_{ee}/2)}}_{\text{Centrifugal F}} - \underbrace{\frac{\partial V_{s2}(R_{dd};1,1)}{\partial R_{dd}}}_{\text{Constraint}} + f(t)$$

Coulomb F

Centrifugal F

Constraint

$$m_d \frac{d^2 \langle R_{dd} \rangle}{dt^2} = - \underbrace{\left\langle \frac{5.26}{R_{dd}^2} \right\rangle}_{(=0)} + 4 \left\langle \frac{m_e v_e^2}{R_{ee}} \right\rangle - \frac{\partial V_{s2}(R_{dd};1,1)}{\partial \langle R_{dd} \rangle} + \langle f(t) \rangle$$

By e-wave averaging

$$m_d \frac{d^2 \langle R_{dd} \rangle}{dt^2} = - \frac{\partial V_{s2}(R_{dd};1,1)}{\partial \langle R_{dd} \rangle} + \langle f(t) \rangle$$

=0 by X-averaging

; = 0 at $R_{dd} = R_{gs} = 74.1 \text{ pm}$; > 0 for $R_{dd} < R_{gs}$, deceleration; < 0 for $R_{dd} > R_{gs}$, acceleration

$R_{dd}(t)$ converges to $R_{dd}(\infty) = R_{gs} = 74.1 \text{ pm}$

Potential of “dede”-type Molecule

with $a_0 = 0.053$ nm (Bohr radius) and $Z = e^+/e$.

We also solved an atomic de^+ system to obtain ground state energy V_h as:

$$V_h = -13.6Z^2/(m_e/m^+). \quad (3.26)$$

For dde^+e^+ molecule state with double electrons or e^+ s, we also extend the solution for $ddee$ given in the text [45] and we have obtained screened potential function $V_{sc^+e^+}$ as:

$$V_{sc^+e^+}(R) = 2V_h + e^2/R + (2J + J' + 2\Delta K + K')/(1 + \Delta^2). \quad (3.27)$$

Here the cross-Coulomb integral J' and cross exchange integral K' are given as:

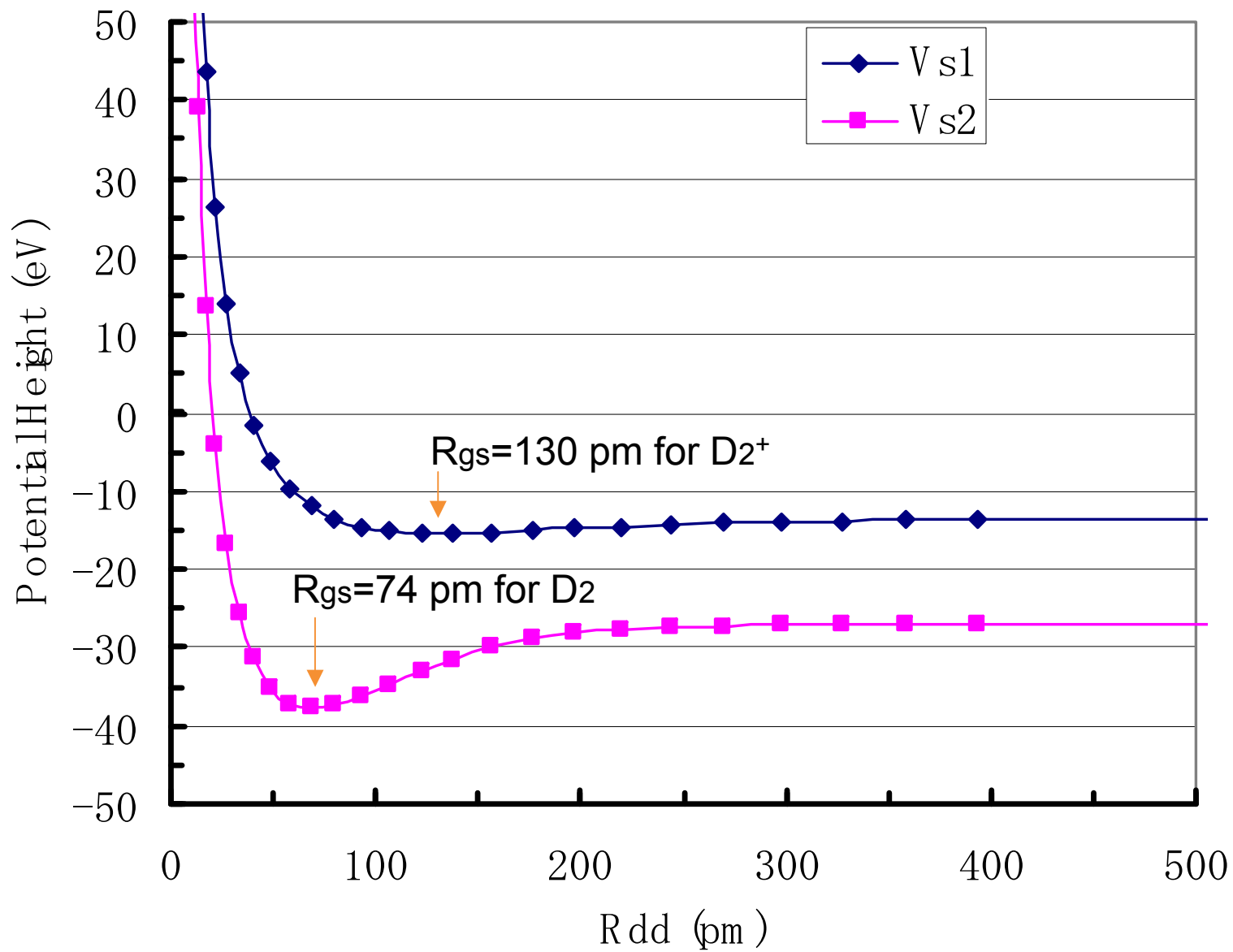
$$J' = (Z^2e^2/a)(1/y - \exp(-2y))(1/y + 11/8 + 3y/4 + y^2/6), \quad (3.28)$$

$$K' = (Z^2e^2/5/a)[- \exp(-2y)(-25/8 + 23y/4 + 3y^2 + y^3/3) + (y/6)((0.5772 + \log y)\Delta^2 + (\Delta')^2E_i(-4y) - 2\Delta\Delta'E_i(-2y))] \quad (3.29)$$

with

$$\Delta' = \exp(-y)(1 - y + y^2/3), \quad (3.30)$$

$$E_i(y) = - \int_0^{\exp(-y)} (1/\log x) dx. \quad (3.31)$$

Trapping Potentials for D₂ and D₂⁺

3.3) D_2^+ (dde) Langevin Equation

$$m_d \frac{d^2 R_{dd}}{dt^2} = -2 \frac{e^2}{R_{de}^2} + \frac{e^2}{R_{dd}^2} + \frac{m_e v_e^2}{R_e} - \frac{\partial V_s(R_{dd}; 1, 1)}{\partial R_{dd}} + f(t)$$

$$-e^2 \left\langle \frac{2}{R_{de}^2} - \frac{1}{R_{dd}^2} \right\rangle + \left\langle \frac{m_e v_e^2}{R_e} \right\rangle = 0$$

$$m_d \frac{d^2 \langle R_{dd} \rangle}{dt^2} = - \frac{\partial V_s(R_{dd}; 1, 1)}{\partial \langle R_{dd} \rangle}$$

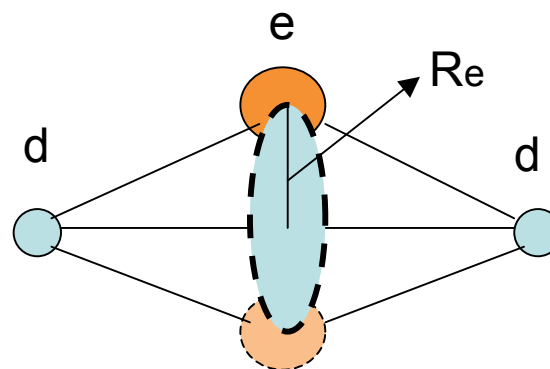
$$\begin{aligned} \langle R_{dd} \rangle(\infty) &= R_{gs} = 138 \text{ pm} \\ \langle H \rangle &= -15.1 \text{ eV} \\ \langle \text{Electron K.E} \rangle &= 13.6 \text{ eV} \end{aligned}$$

$$\langle R_{dd} \rangle = 138 \text{ pm}$$

$$\langle R_{de} \rangle = 86.9 \text{ pm}$$

$$\langle R_e \rangle = 52.9 \text{ pm}$$

Elongated Di-Cone



Potential for “dde”-type Molecule

Screened potentials $V_s n(m^*/m_e, e^*/e)(R)$ were calculated for dde^{*} molecular states using extended solutions for dde state given in a text of quantum mechanics by the well-known technique of variational method [45] as:

$$\underline{V_s(m^*/m_e, e^*/e)(R)} = V_h + e^2/R + (J + K)/(1 + \Delta), \quad (3.20)$$

where the Coulomb integral J , the exchange integral K , and the non-orthogonal integral Δ are given as [45]:

$$J = Z(e^2/a)[-1/y + (1 + 1/y) \exp(-2y)], \quad (3.21)$$

$$K = -Z(e^2/a)(1 + y) \exp(-y), \quad (3.22)$$

$$\Delta = (1 + y + y^2/3) \exp(-y). \quad (3.23)$$

With

$$Y = R/a, \quad (3.24)$$

$$a = a_0/Z/(m^*/m_e) \quad (3.25)$$

4. QM-Average for Complex D-cluster under Platonic Symmetry

- Average on Electron-wave function is replaced with Friction (Constraint) as

$$\langle \text{Constraint} \rangle_{\text{electron-wave}} = -N_f \frac{\partial V_{si}(R_{dd}; 1, 1)}{\partial R_{dd}}$$

N_f : Number of faces for Platonic polyhedron

V_{si} : D_2 ($i=2$) or D_2^+ ($i=1$) trapping potential

- Average on d-d wave function: using

$$\Psi(R, R') = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(R'-R)^2}{2\sigma^2})$$

QM Average of Langevin Equation: for $N_e > 2$

$$N_e m_d \frac{d^2 R}{dt^2} = -\frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + f(t)$$

N_e : Number of d-d edges

$$N_e m_d \left\langle \Psi(R, R') \left| \frac{d^2 R}{dt^2} \right| \Psi(R, R') \right\rangle = - \left\langle \Psi(R, R') \left| \frac{k}{R^2} \right| \Psi(R, R') \right\rangle$$

$$- N_f \left\langle \Psi(R, R') \left| \frac{\partial V_s}{\partial R} \right| \Psi(R, R') \right\rangle + \left\langle \Psi(R, R') \left| f(t) \right| \Psi(R, R') \right\rangle$$

$$\Psi(R, R') = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(R'-R)^2 / (2\sigma^2))$$

Gaussian Wave Function

$$N_e m_d \frac{d^2 \langle R \rangle}{dt^2} = -\frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + \langle f(t) \rangle$$

Equation for
Expectation Value

$$N_e m_d \frac{d^2 R}{dt^2} = - \frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + f(t)$$

Centripetal
Coulomb Force

Friction by
Electron Cloud

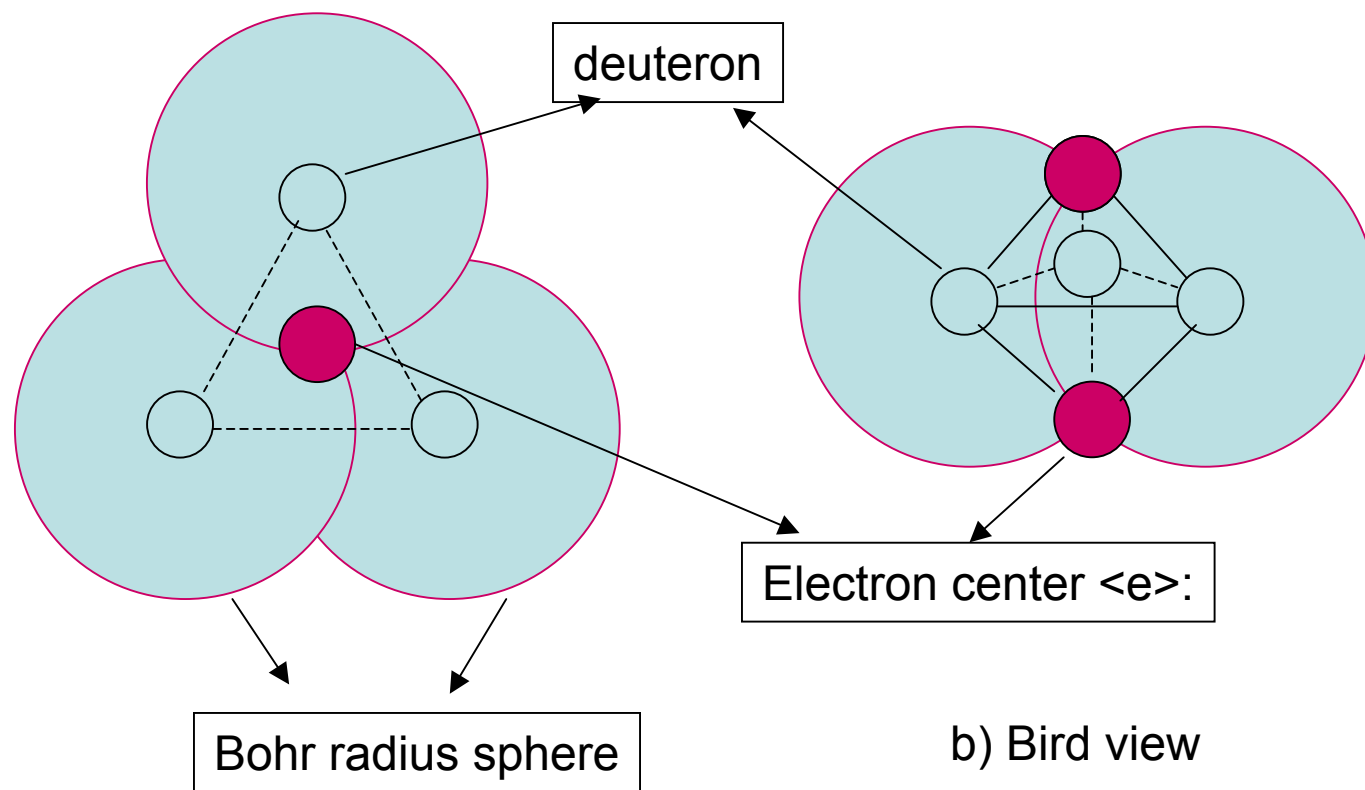
QM Fluctuation
Of Force

Table-1: parameters of D-cluster Langevin equation

cluster	N_e : Number of d-d edges	k: Total Coulomb Force parameter (keVpm)	Type of electron trapping potential on a surface	N_f : number of faces
D_2	1	0	$i = 2$	1
D_2^+	1	0	$i = 1$	1
D_3^+	3	6.13	$i = 1$	6
4D/TSC	6	11.85	$i = 2$	6
6D ²⁻ /OSC	12	29.3	$i = 1$	24

4.2) Application to Tri-Atomic Molecule

3D⁺ Ion ; Semi-classical view of particle arrangement



a) Top View

b) Bird view

Trigonal Dipyramid
: Triangle-Dipole Coupling

3D⁺ Molecule

- Coulomb Energy:

$$E_C = -6 \frac{e^2}{R_{de}} + 3 \frac{e^2}{R_{dd}} + \frac{e^2}{R_{ee}}$$

For Platonic arrangement:

$$E_C = -\left(6\sqrt{2} - 3 - \frac{\sqrt{6}}{2}\right) \frac{e^2}{R_{dd}} = -\frac{6.13}{R_{dd}}$$

D_3^+ Ion Langevin Equation

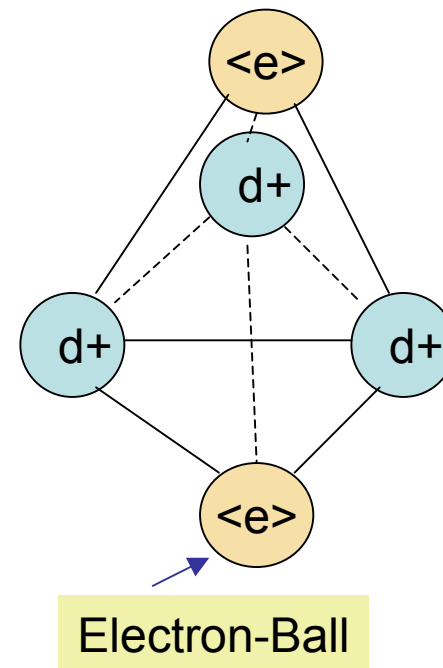
$$3m_d \frac{d^2 R_{dd}}{dt^2} = -\frac{6.13}{R_{dd}^2} - 6 \frac{\partial V_s(R_{dd}; 1, 1)}{\partial R_{dd}} + f(t)$$

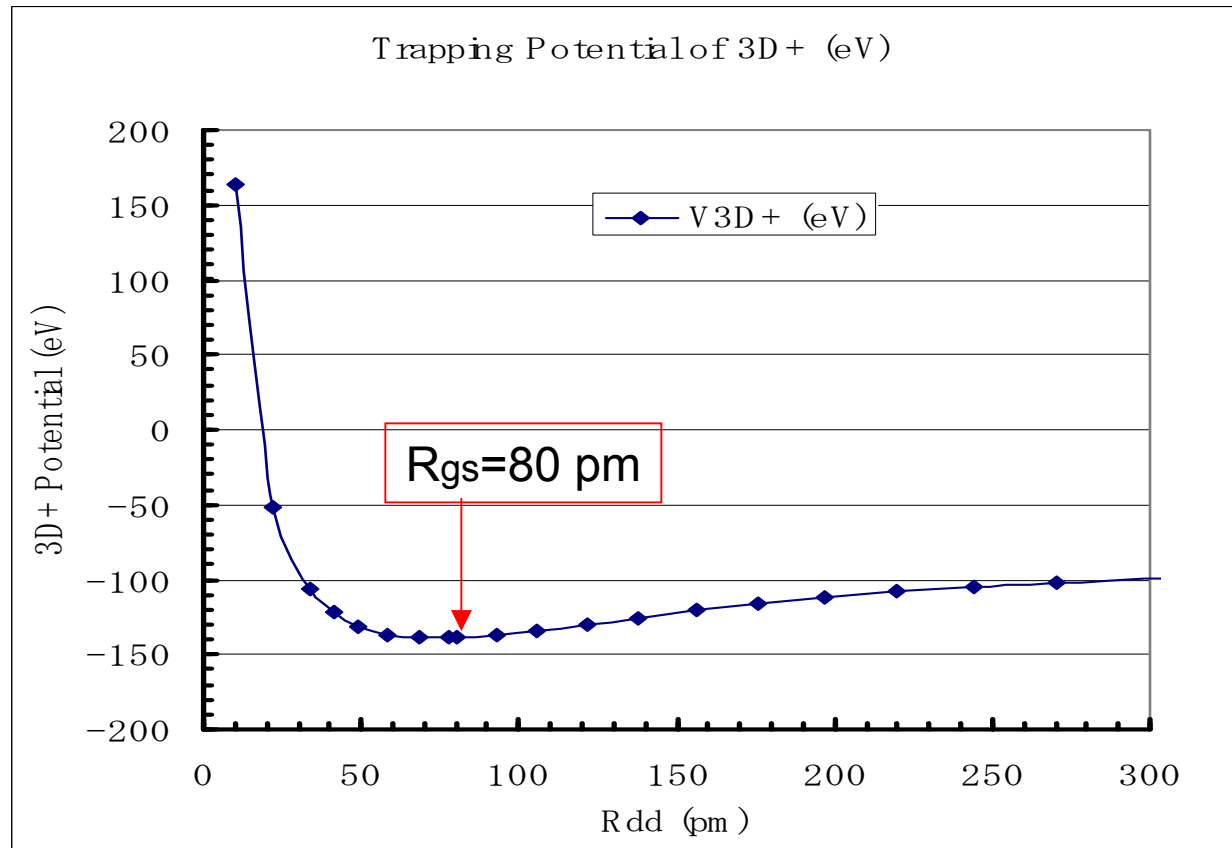
Coulomb F

Constraint
6 dde faces

$$V_{3D+main}(R_{dd}) = -\frac{6.13}{R_{dd}} + 6V_s(R_{dd}; 1, 1)$$

3D+ Trapping Potential

+ $\langle f(t) \rangle$ biasRatio = $6.13/6 = 1.02$ CF: 1.975 for 4D/TSC



$$3m_d \frac{d^2 \langle R_{dd} \rangle}{dt^2} = -\frac{6.13}{\langle R_{dd} \rangle^2} - 6 \frac{\partial V_s(R_{dd}; 1, 1)}{\partial \langle R_{dd} \rangle} + \langle f(t) \rangle$$

$$\langle f(t) \rangle = \left\langle -\frac{\partial \Delta E_c}{\partial R'} X^2(R_{dd}; \sigma, t) \right\rangle$$

:Distortion of Coulomb force from 3D
Regular triangle arrangement (about 30%)

Summary of Known D-molecules

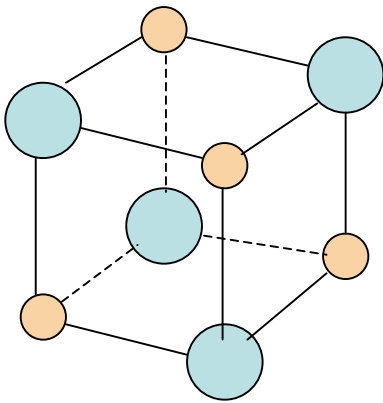
- Known D-molecules are regarded as Platonic Symmetric System with orthogonal coupling of electron wave function and deuteron wave function
- Due to the Platonic symmetry, 3-dim. Dynamics can be treated by 1-dim. Dynamics using Langevin equation
- Deuteron Trapping Potential is estimated from individual Langevin equation

5. Now the Methodology is applied for 4D/TSC and 6D/OSC

- Formulate the central condensation force (CCF) by Coulombic interaction
- Formulate the fluctuation force term by change of CCF in deformed state from the Platonic symmetry
- Define cluster trapping potential (time-dependent)
- Execute numerical calculation

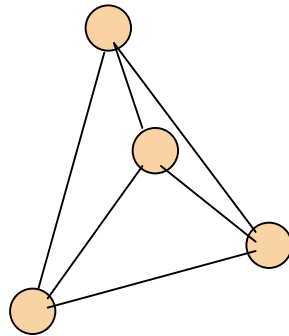
Double Platonic Arrangement : Tetrahedron-Tetrahedron Coupling

a) TSC



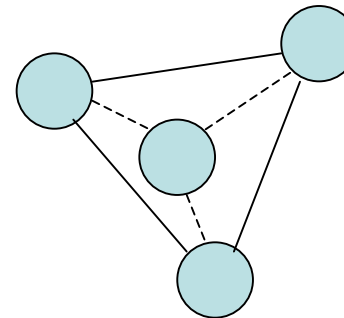
12 Attractive
Coulomb forces
Between d-e pairs
on 6 surfaces
And 4 Attractive
Forces between
4 diagonal d-e pairs

b) Electron
tetrahedron



6 repulsive
Coulomb
Forces
Between
electrons

c) Deuteron
tetrahedron



6 repulsive
Coulomb
Forces
Between
deuterons

Coulomb Energy of TSC

- System Coulomb Energy for CCF

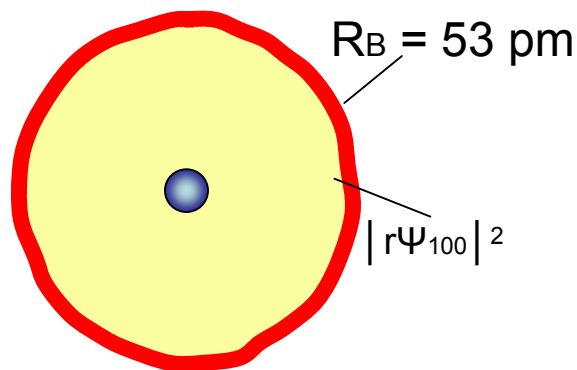
$$\langle E_{C-TSC} \rangle = 12\left(-\frac{e^2}{R_{de}}\right) + 12\left(\frac{e^2}{R_{dd}}\right) + 4\left(-\frac{e^2}{\sqrt{3}R_{de}}\right) = -\frac{8.38}{R_{de}}$$

In keV unit with R in pm unit

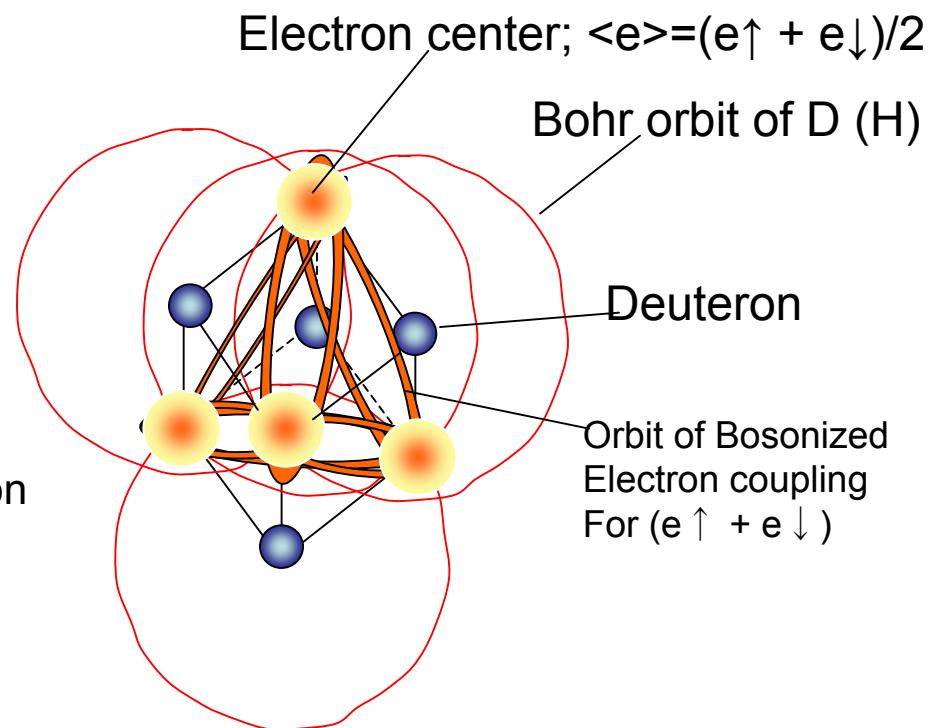
$$R_{dd} = \sqrt{2}R_{de}$$

$$\langle E_{k,e-ball} \rangle = 4\left(\frac{1}{2}m_e v_d^2\right) = 4\left(\frac{m_e}{M_d}\right)E_d \leq 0.88eV$$

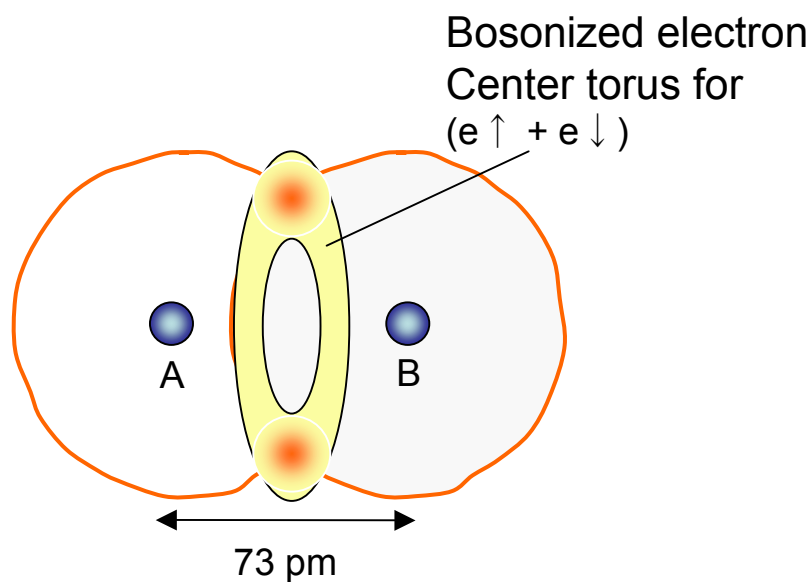
Feature of QM Electron Cloud



a) D atom (stable)



c) 4D/TSC (life time about 60 fs)



b) D₂ molecule (stable): $\Psi_{2D} = (2+2\Delta)^{-1/2} [\Psi_{100}(r_{A1}) \Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2}) \Psi_{100}(r_{B1})] X_s(S1, S2)$

Wave Function for 4D/TSC (t=0)

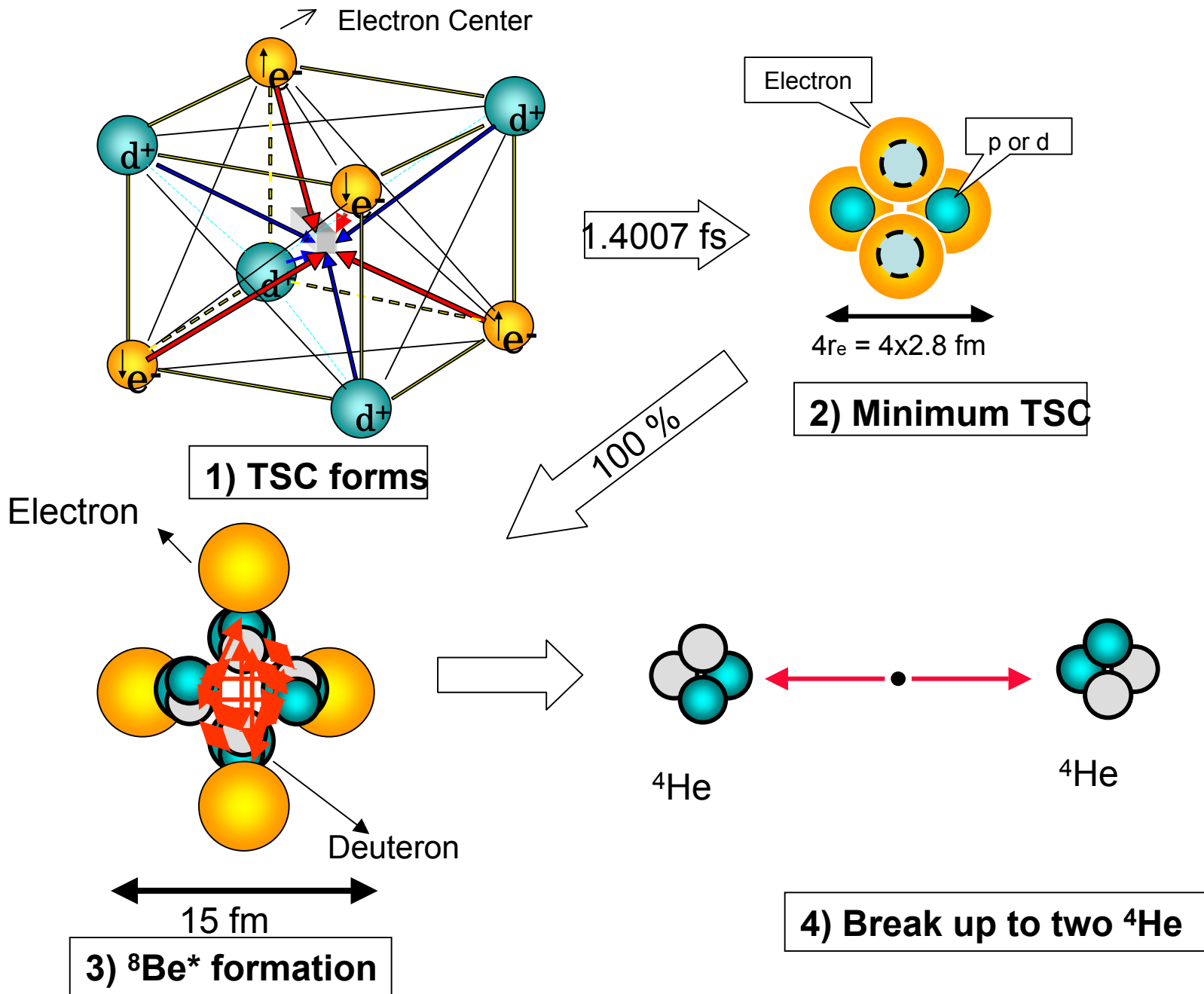
- $\Psi_{4D} \sim a_1 [\Psi_{100}(r_{A1}) \Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2}) \Psi_{100}(r_{B1})] X_s(S1, S2)$
+ $a_2 [\Psi_{100}(r_{A1}) \Psi_{100}(r_{D4}) + \Psi_{100}(r_{A4}) \Psi_{100}(r_{D1})] X_s(S1, S4)$
+ $a_3 [\Psi_{100}(r_{A2}) \Psi_{100}(r_{C4}) + \Psi_{100}(r_{A4}) \Psi_{100}(r_{C2})] X_s(S2, S4)$
+ $a_4 [\Psi_{100}(r_{B1}) \Psi_{100}(r_{D3}) + \Psi_{100}(r_{B3}) \Psi_{100}(r_{D1})] X_s(S1, S3)$
+ $a_5 [\Psi_{100}(r_{B2}) \Psi_{100}(r_{C3}) + \Psi_{100}(r_{B3}) \Psi_{100}(r_{C2})] X_s(S2, S3)$
+ $a_6 [\Psi_{100}(r_{C3}) \Psi_{100}(r_{D4}) + \Psi_{100}(r_{C4}) \Psi_{100}(r_{D3})] X_s(S3, S4)$

6-Bonds of “Bosonized” electron-pairs ($e\uparrow + e\downarrow$), which forms
Regular Tetrahedron (PA)

4-Electron-Centers at Vertexes of **Regular Tetrahedron (PA)**

$$u_{1s1}(r) = \Psi_{100}(r) = (1/\pi)^{1/2} (1/a_B)^{3/2} \exp(-r/a_B)$$

Result of Dynamic Condensation of 4D/TSC by Langevin Equation



5.2 4D/TSC Langevin Equation for Monte-Carlo Calculation

$$6m_d \frac{d^2 R_{dd}(t)}{dt^2} = -\frac{11.85}{[R_{dd}(t)]^2} - 6 \frac{\partial V_{s2}(R_{dd}(t); 1, 1)}{\partial R_{dd}(t)} + \langle f(t) \rangle + f'(t)$$

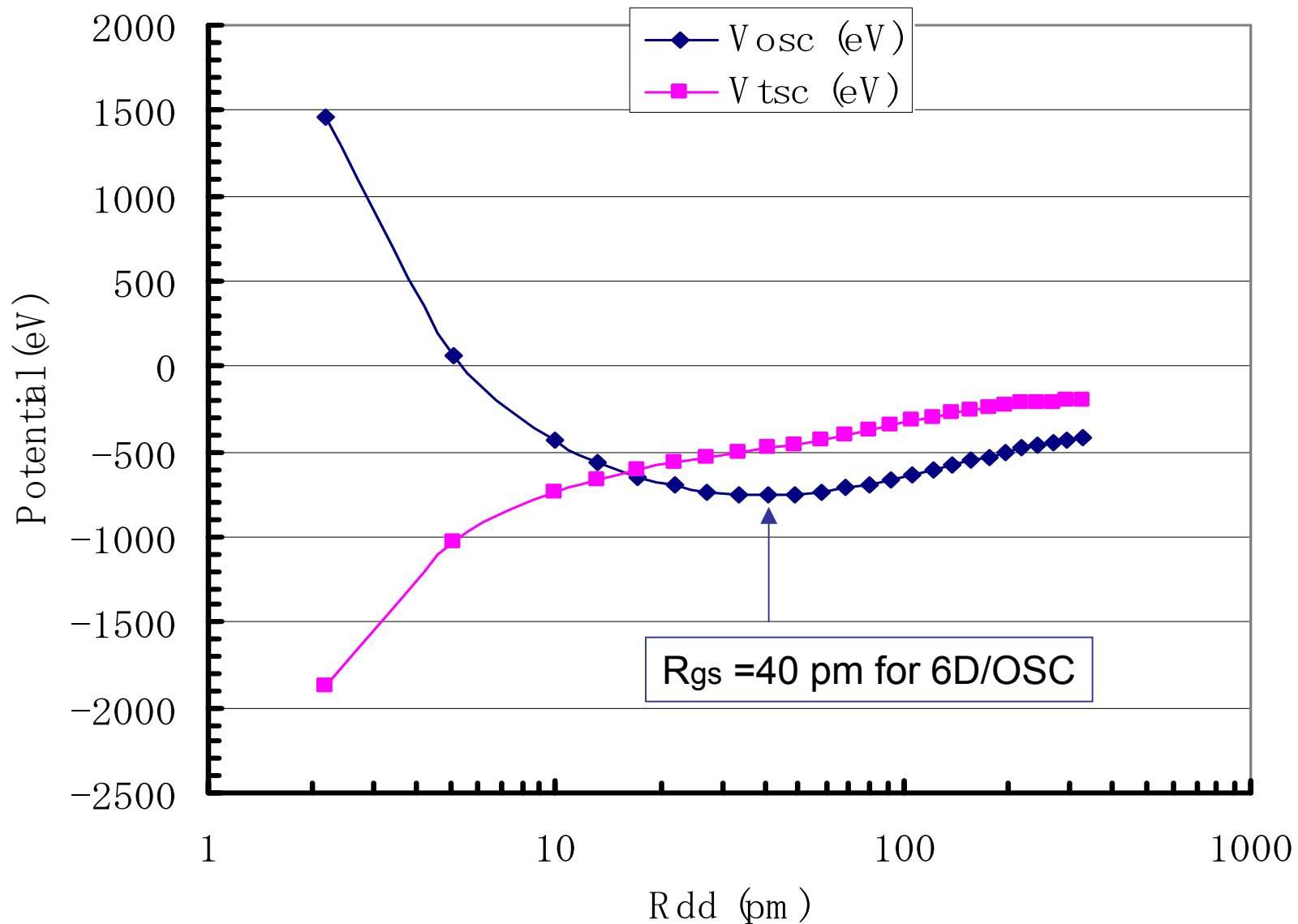
$$f'(t) = f(t) - \langle f(t) \rangle$$

$$V_{tsc-main} = -\frac{11.85}{R_{dd}} + 6V_{s2}(R_{dd}; 1, 1)$$

$$f(t) = \left[-\frac{\partial \Delta E_c(R_{dd})}{\partial R_{dd}} \right] \text{mod} [X^2(R'_{dd}; R_{dd}(t))]$$

$$X^2(R'_{dd}; R_{dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(R'_{dd} - R_{dd}(t))^2 / (2\sigma^2)]$$

Main Trapping Potential of 4D/TSC and 6D/OSC



Langevin Equation for Total TSC System with Maximum-Estimated Friction

$$6m_d \frac{d^2 R_{dd}(t)}{dt^2} = -BA \times \frac{11.85}{[R_{dd}(t)]^2} - 6 \frac{\partial V_s(R_{dd}(t); m, Z)}{\partial R_{dd}(t)} + f(t)$$

Main Condensation Force

6 d-d edges of
Deuteron tetrahedron

6 dede surfaces of TSC
Constraint by electron waves

QM-fluctuation of deuterons

Always Acceleration for $R_{dd} > 10$ fm

BA: parameter (< 1.0) to merge $\langle f(t) \rangle$ bias

Verlet's Method

$$G(r, t) = \frac{1.975}{m_d [R(0) - r(t)]^2} + \frac{1}{m_d} \frac{\partial V_s(R_{dd}; m, Z)}{\partial R_{dd}}$$

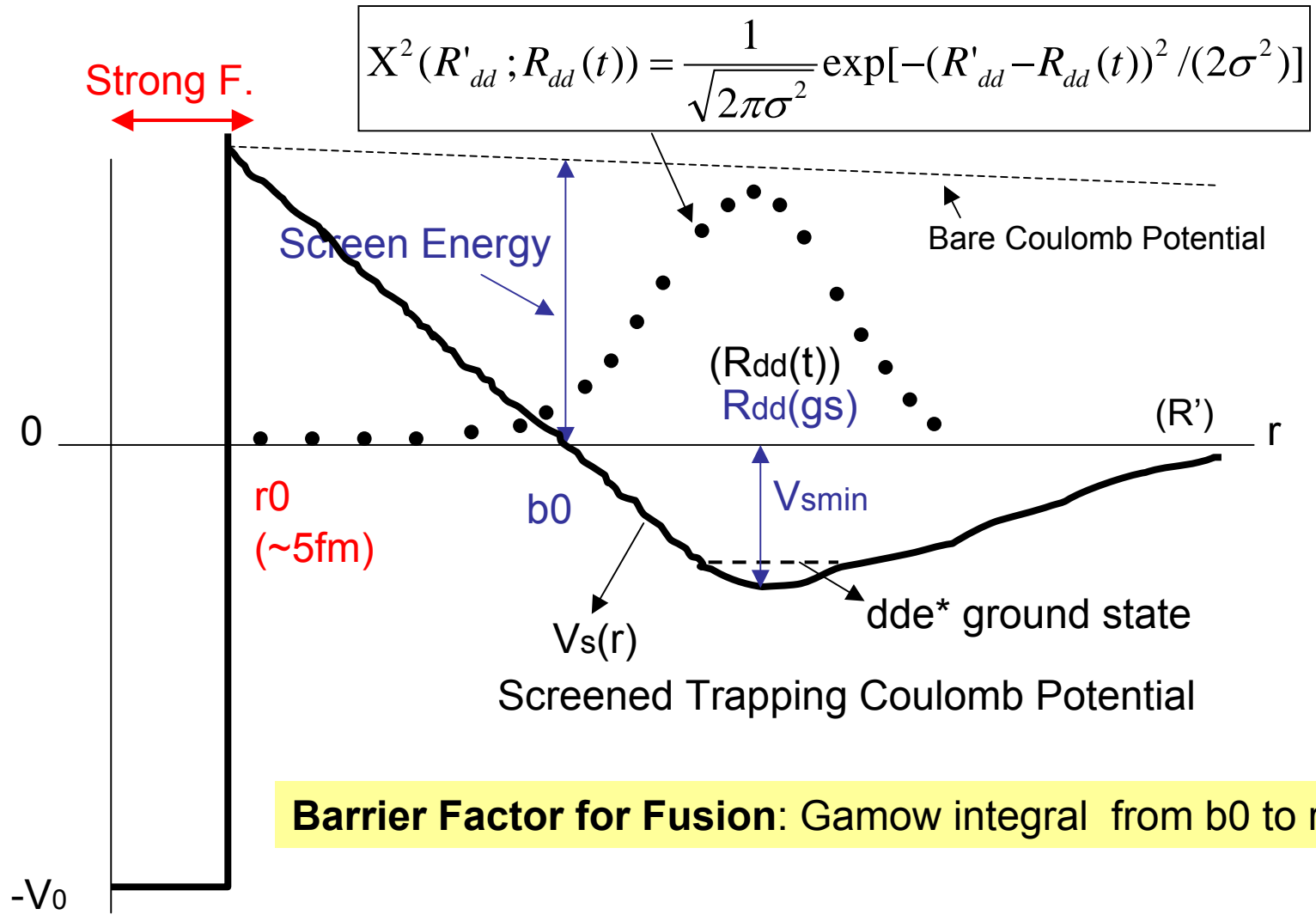
$$R_{dd}(t) = R_0 - r(t)$$

$$\frac{d^2 r(t)}{dt^2} = G(r, t)$$

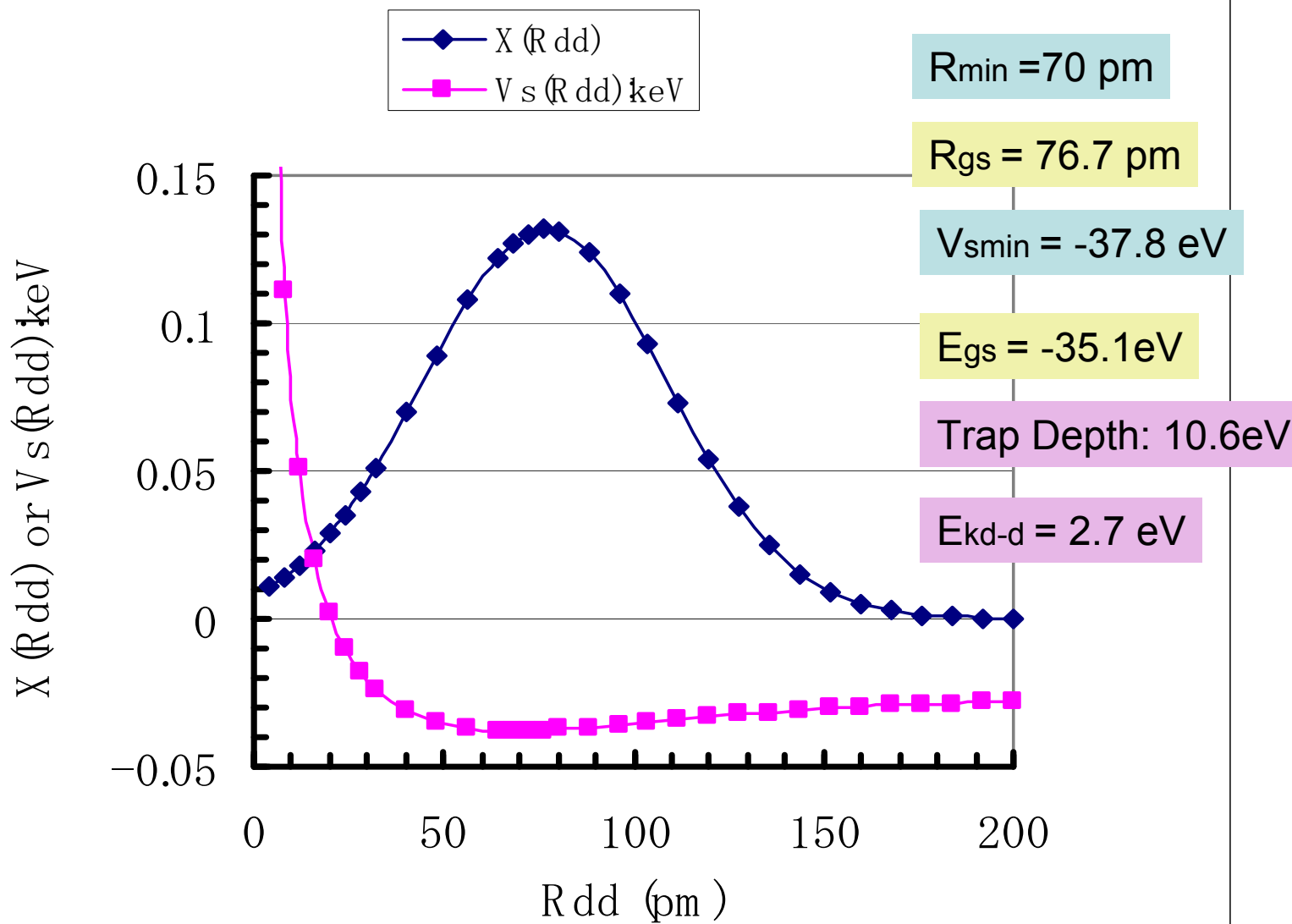
$$r(t + \Delta t) = r(t) + v(t)\Delta t + \frac{1}{2} G(r, t)(\Delta t)^2$$

$$v(t + \Delta t) = v(t) + \frac{\Delta t}{2} [G(r, t + \Delta t) + G(r, t)]$$

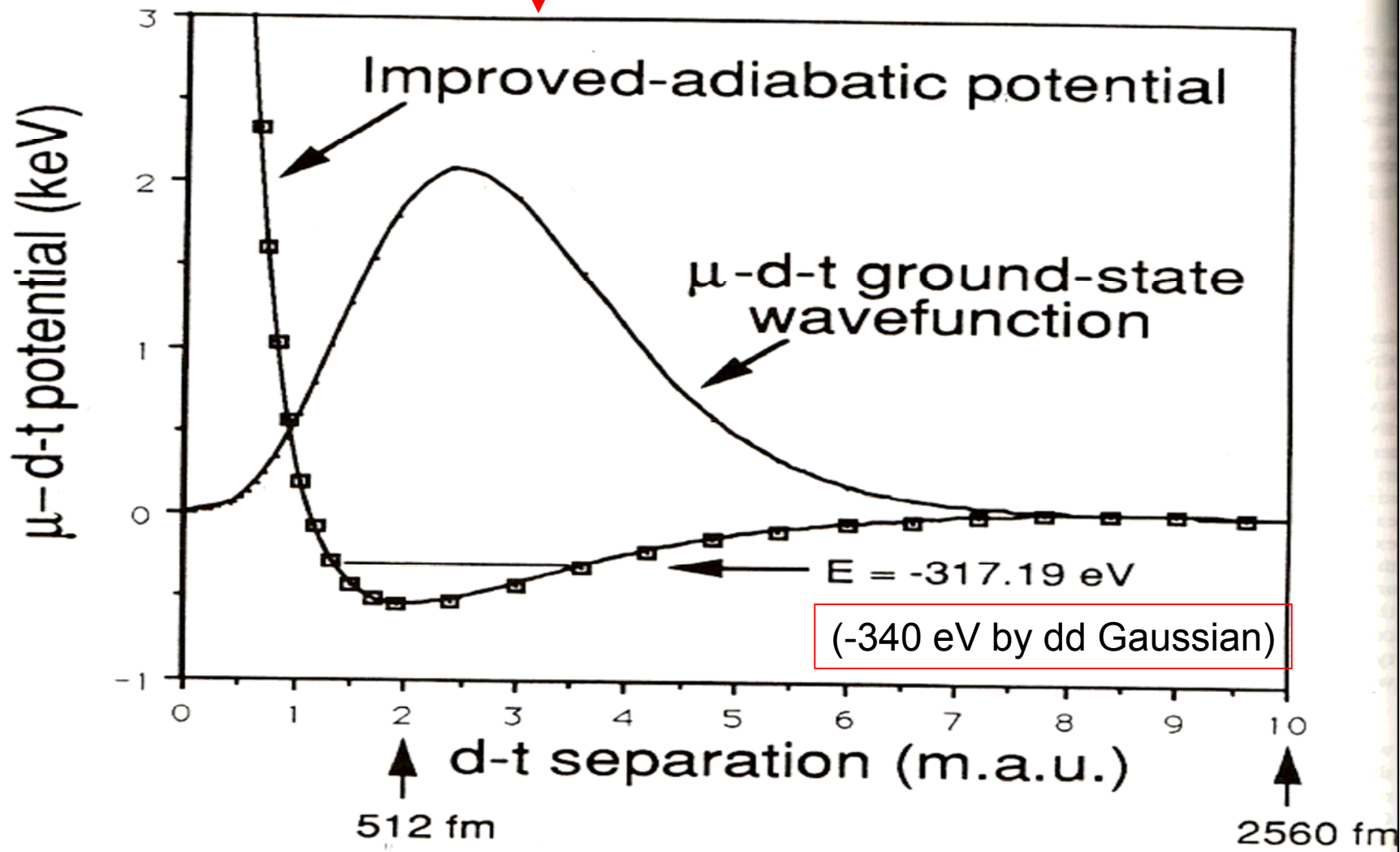
Adiabatic Potential for Molecule dde^* and its ground state squared wave function



D2 Molecule Potential and Gaussian Wave Function
calculated by GWF2 Code with $\sigma/R_{gs}=0.3$

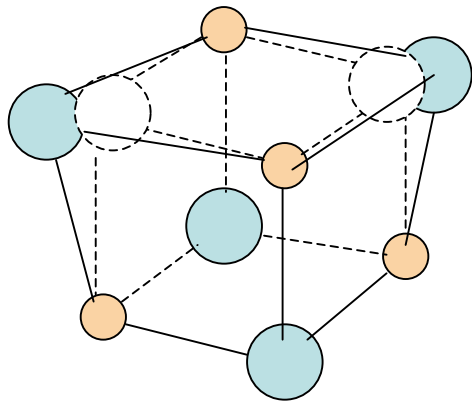


$R_{gs} = 805$ fm for dd Gaussian WF

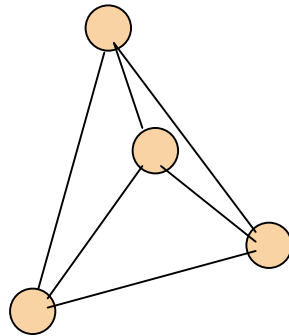


Distortion of Double Platonic Arrangement

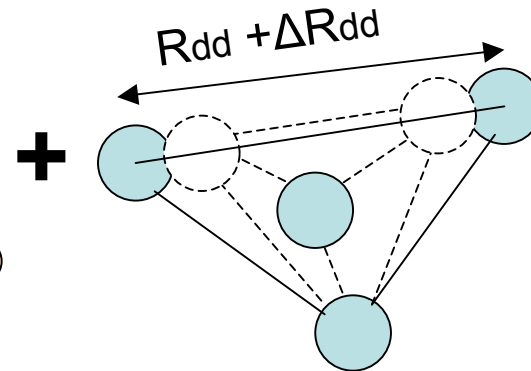
a) TSC



b) Electron tetrahedron



c) Deuteron tetrahedron



12 Attractive
Coulomb forces
Between d-e pairs
on 6 surfaces
And 4 Attractive
Forces between
4 diagonal d-e pairs

6 repulsive
Coulomb
Forces
Between
electrons

6 repulsive
Coulomb
Forces
Between
deuterons

Exercise

$$\begin{aligned} -\frac{\partial \Delta E_c}{\partial R} &\approx -k \frac{1}{2} \left(\frac{1}{(R + \Delta R)^2} + \frac{1}{(R - \Delta R)^2} \right) + k \frac{1}{R^2} \\ &\approx -\frac{k}{2R^2} \left(1 - \frac{2\Delta R}{R} - \left(\frac{\Delta R}{R} \right)^2 + 1 + \frac{2\Delta R}{R} - \left(\frac{\Delta R}{R} \right)^2 \right) + \frac{k}{R^2} \\ &= \frac{k}{R^2} \left(\frac{\Delta R}{R} \right)^2 \end{aligned}$$

TSC Langevin Step2

$$6m_d \frac{d^2 R_{dd}(t)}{dt^2} = -BA \times \frac{11.85}{[R_{dd}(t)]^2} - 6 \frac{\partial V_s(R_{dd}(t); m, Z)}{\partial R_{dd}(t)} + f(t)$$

$$f(t) = \left[-\frac{\partial \Delta E_c(R_{dd})}{\partial R_{dd}} \right] \text{mod} [X^2(R'_{dd}; R_{dd}(t))]$$

$$f(t)=0 \text{ for } R'_{dd}=R_{dd}$$

$$X^2(R'_{dd}; R_{dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(R'_{dd}-R_{dd}(t))^2 / (2\sigma^2)]$$

$$\sigma = 0.372 R_{dd}(t)$$

$$\Delta R_{dd} = R'_{dd} - R_{dd}(t)$$

$$-\frac{\partial \Delta E_c(R'_{dd})}{\partial R_{dd}} = 6.60 \frac{[\Delta R_{dd}]^2}{[R_{dd}(t)]^4}$$

Change of TSC Coulomb Energy by Distortion of Platonic Symmetry

$$f(t) = f_{up}(t) + f_{down}(t)$$

Fluctuation of 2 d-d pairs of TSC

Averaged Treatment of $\langle f(t) \rangle$

$$\langle f(t) \rangle = 2 \times 6.6 \int_0^\infty \frac{[R' - R_{dd}(t)]^2}{[R_{dd}(t)]^4} X^2(R') dR'$$

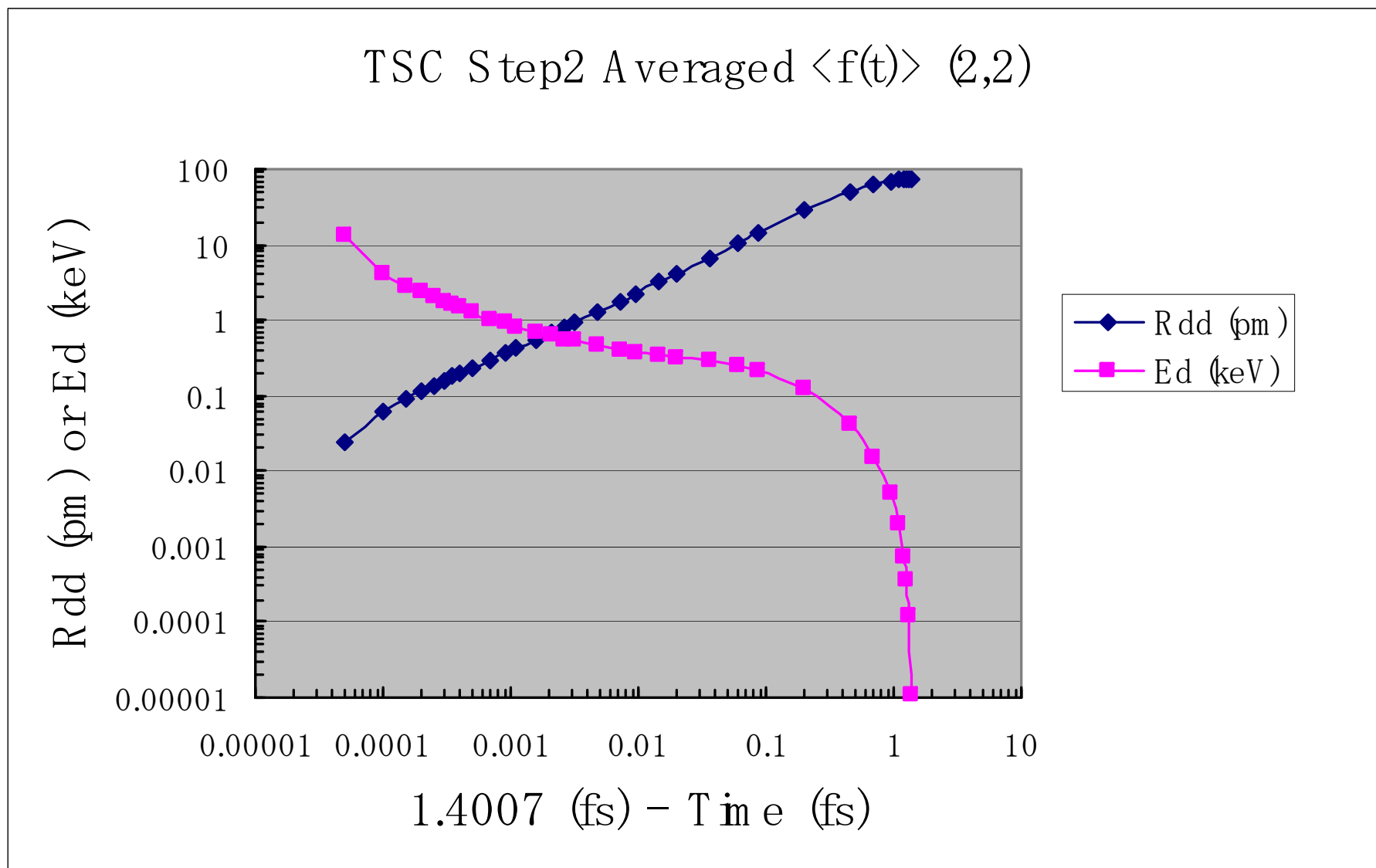
$$\langle f(t) \rangle = \frac{13.2\sigma^2}{\sqrt{\pi}[R_{dd}(t)]^4} \int_0^\infty \sqrt{x} \exp(-x) dx = 13.2 \frac{\sigma^2}{[R_{dd}(t)]^4}$$

$$\sigma = 0.372 R_{dd}(t)$$

$$6m_d \frac{d^2 \langle R_{dd} \rangle}{dt} = -BA \frac{11.85}{\langle R_{dd} \rangle^2} - 6 \frac{\partial V_s(R_{dd}; m, Z)}{\partial \langle R_{dd} \rangle}$$

$$BA = 0.846$$

BA=0.846



$E_d = 13.68$ keV at $R_{dd} = 24.97$ fm, with $V_{trap} = -130.4$ keV

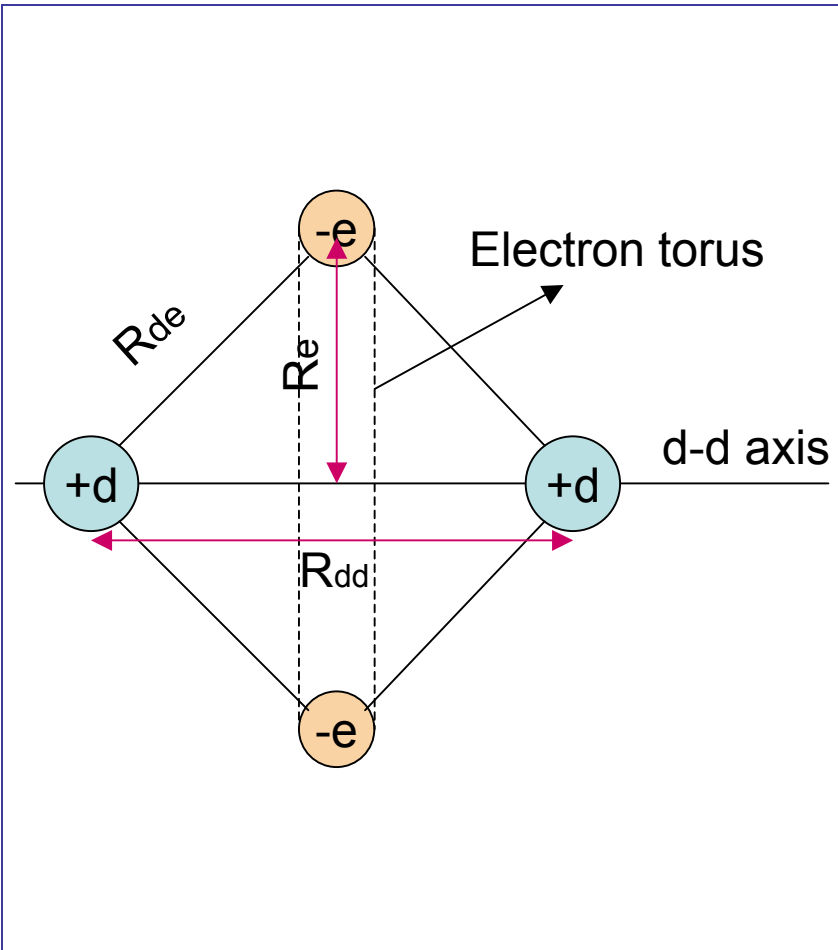
5.3 Time Dependent dd Trapping Potential of TSC

$$6m_d \frac{d^2 \langle R_{dd} \rangle}{dt^2} = -\frac{11.85}{\langle R_{dd} \rangle^2} - 6 \frac{\partial V_s(\langle R_{dd} \rangle; m, Z)}{\partial \langle R_{dd} \rangle} + 6.6 \left\langle \frac{(R' - R_{dd})^2}{R_{dd}^4} \right\rangle$$

Approximate TSC Trapping Potential is given as

$$V_{tsc}(R': R_{dd}(t)) = -\frac{11.85}{R_{dd}(t)} + 6V_s(R_{dd}(t); m, Z) + 2.2 \frac{|R' - R_{dd}(t)|^3}{[R_{dd}(t)]^4}$$

Mean Particle Kinetic Energies of the “dede” System of TSC



$$(e^2/R_e^2) = (m_e v_e^2/R_e) = (2E_{ke}/R_e)$$

$$R_e = R_{dd}/2$$

$$E_{ke} = 1.44/R_{dd} : [\text{keV}] \text{ by } R \text{ in pm}$$

At $R_{dd}=0.025$ pm (25 fm)

$$E_{ke} = 57.6 \text{ keV}$$

$$E_{kd-d} = 13.68 \text{ keV}$$

$$V_{tsc-min} = -130.4 \text{ keV}$$

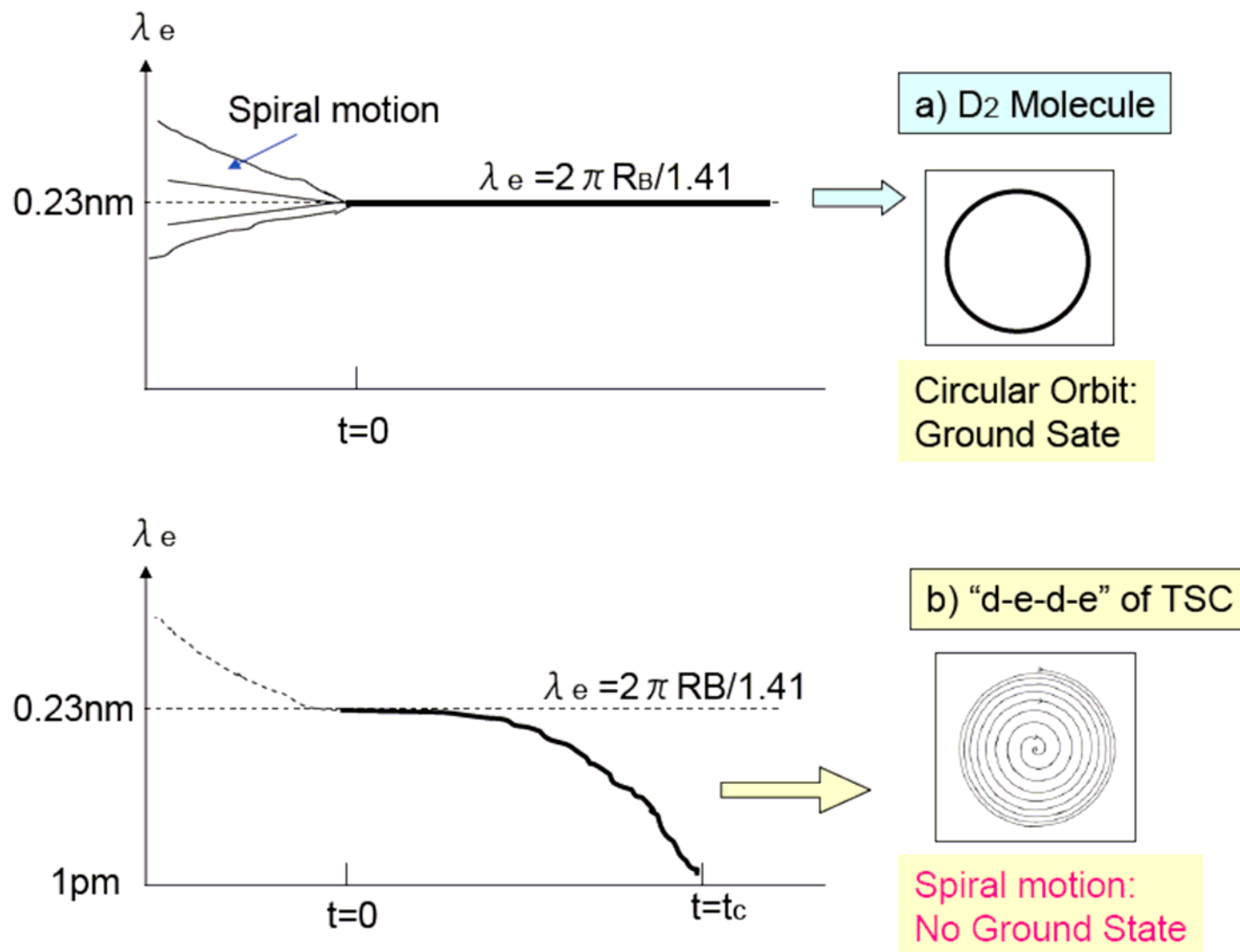
Orbit of Electron Center: a) D₂ Molecule, b) TSC

Fig.4: Time dependent behavior of effective electron wave length, a) D₂ molecule, b) "d-e-d-e" EQPET molecule of 4D/TSC

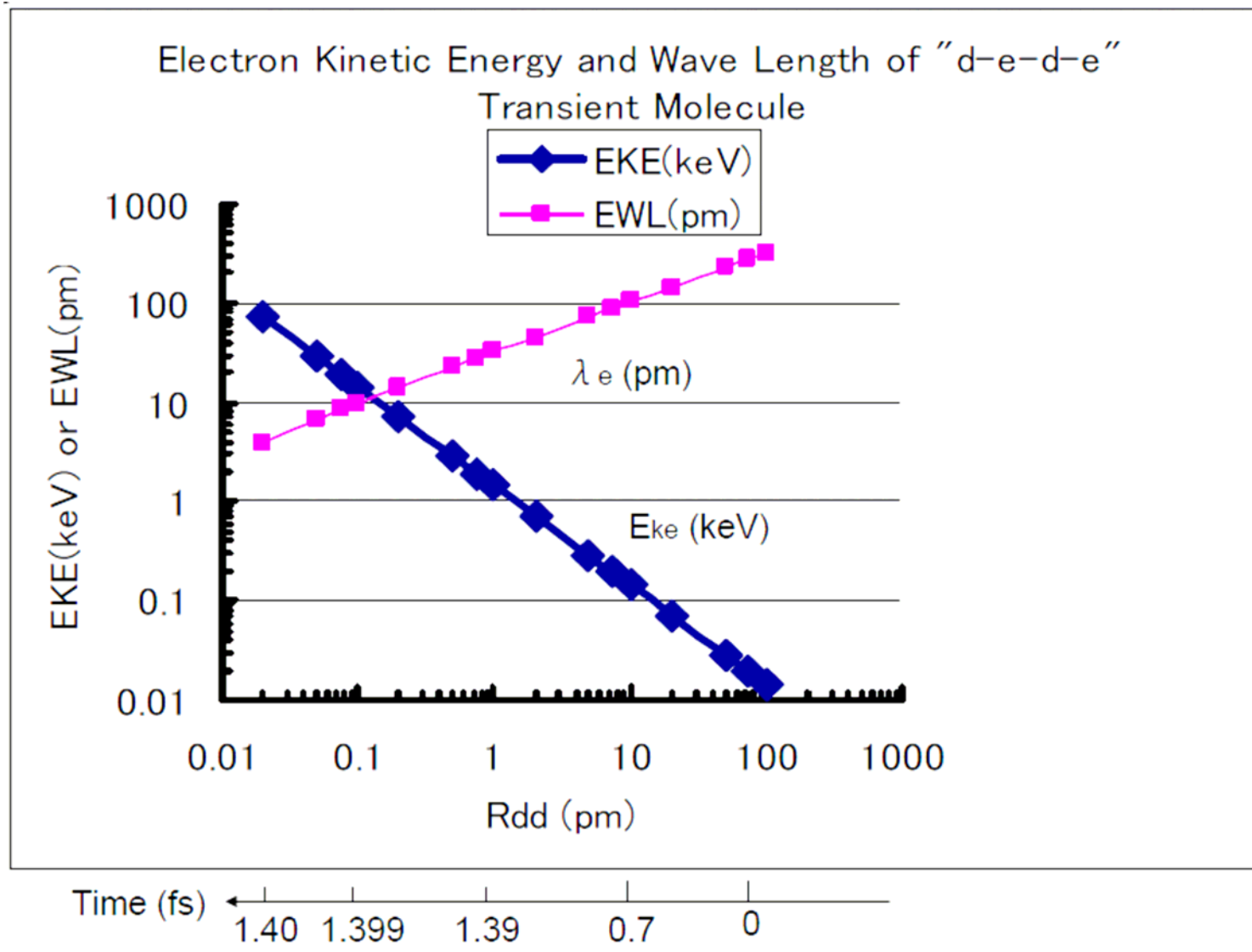
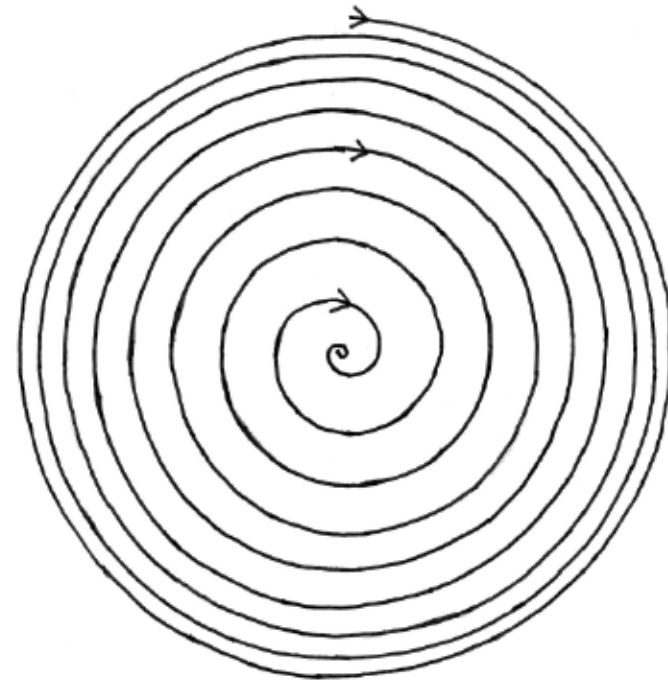
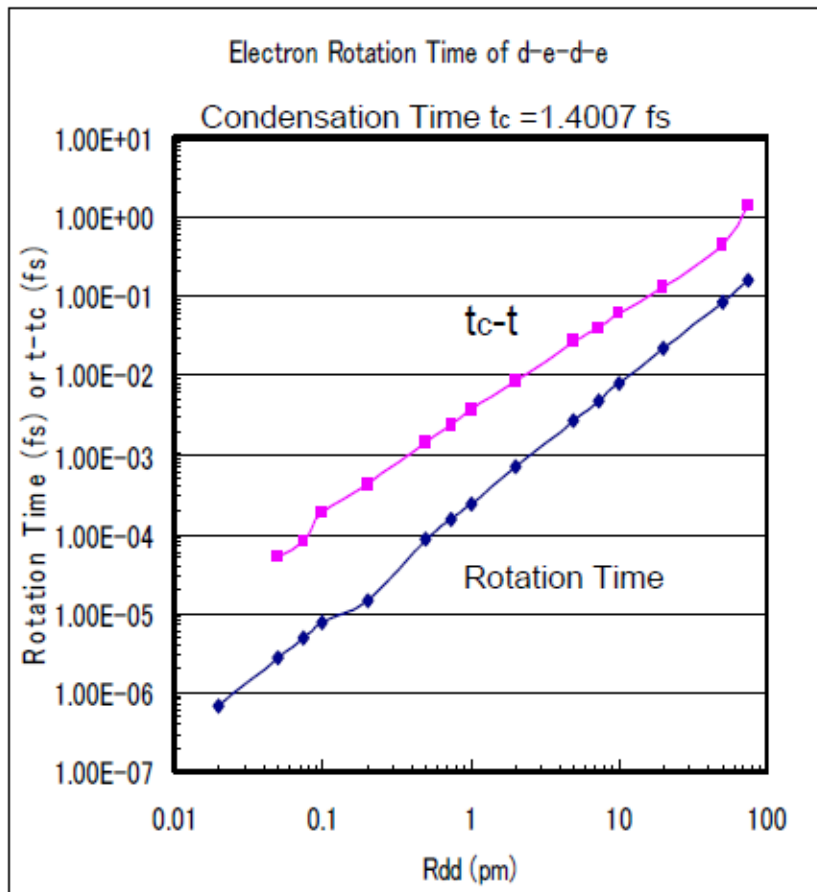


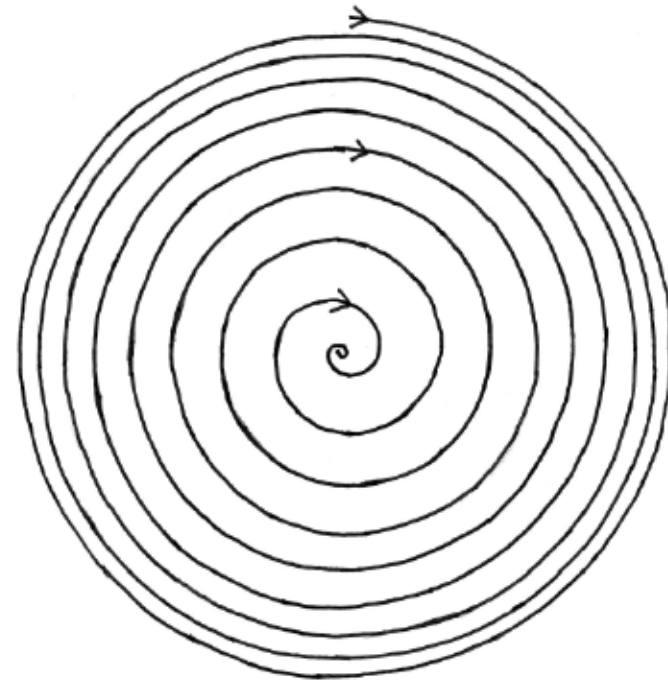
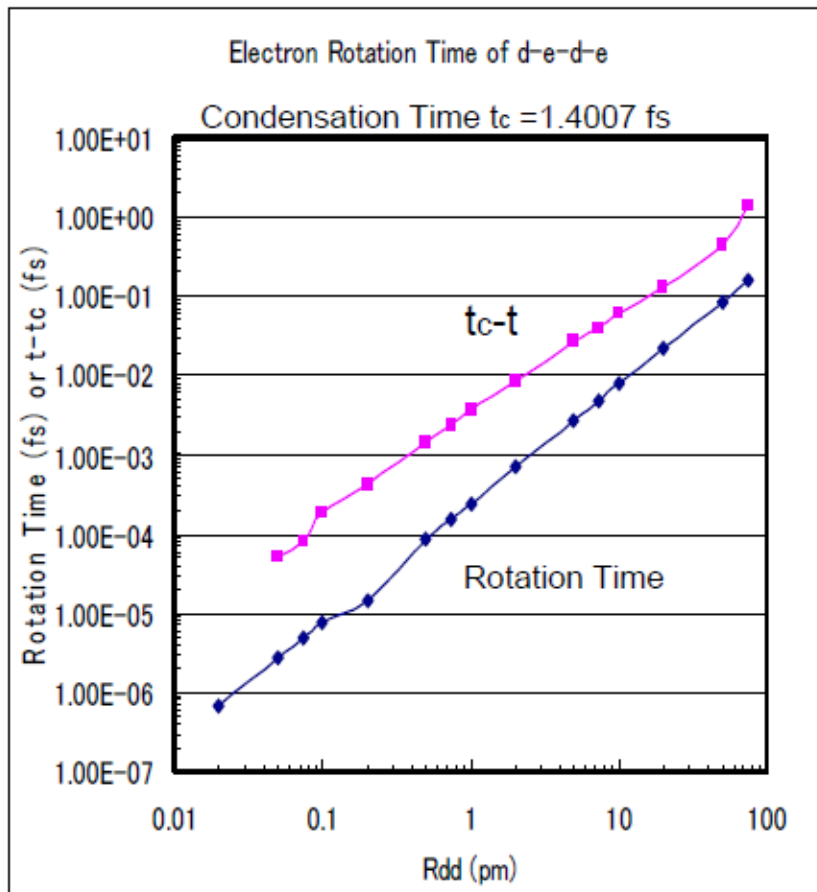
Fig.6: Time variation of mean electron kinetic energy (EKE) and its wave length (EWL) during the 4D/TSC condensation motion in 1.4007fs condensation time.



a) Mean rotation time of electron in “d-e-d-e” of TSC, mean rotation number is about 6 at every R_{dd} step

b) Feature of spiral motion of electron in 1.4007 fs condensation time interval

Fig.7: Mean rotation time of electron cloud center under 4D/TSC condensation motion (left) and expanded feature of electron spiral motion (right)



a) Mean rotation time of electron in “d-e-d-e” of TSC, mean rotation number is about 6 at every R_{dd} step

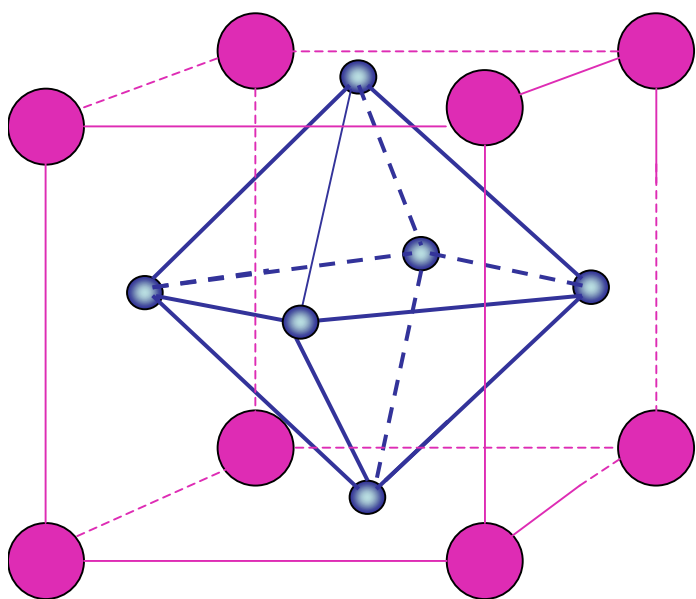
b) Feature of spiral motion of electron in 1.4007 fs condensation time interval

Fig.7: Mean rotation time of electron cloud center under 4D/TSC condensation motion (left) and expanded feature of electron spiral motion (right)

5.4 Application to 6D/OSC

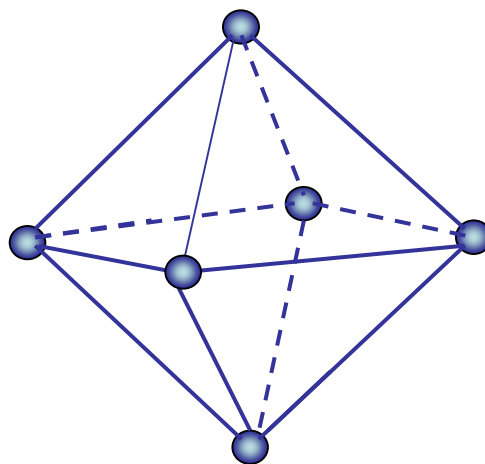
Triaxis Octahedron

$$D_6^{2-} : 6D/OSC = \text{6D Octahedron} + \text{8e Hexahedron}$$



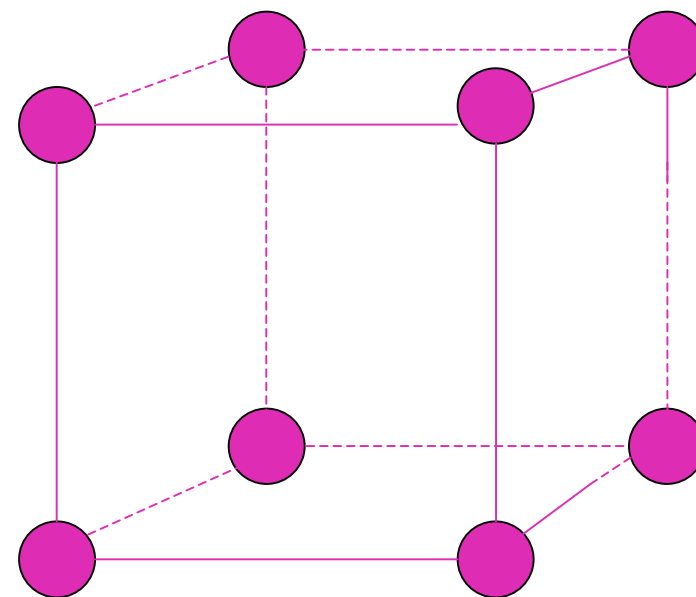
$$\begin{aligned} & -24(e^2/R_{de}) \\ & -24(e^2/R_{d'e}) \end{aligned}$$

$$R_{de} = 0.707R_{dd}$$



$$\begin{aligned} & 12(e^2/R_{dd}) \\ & + 4e^2/(1.414R_{dd}) \end{aligned}$$

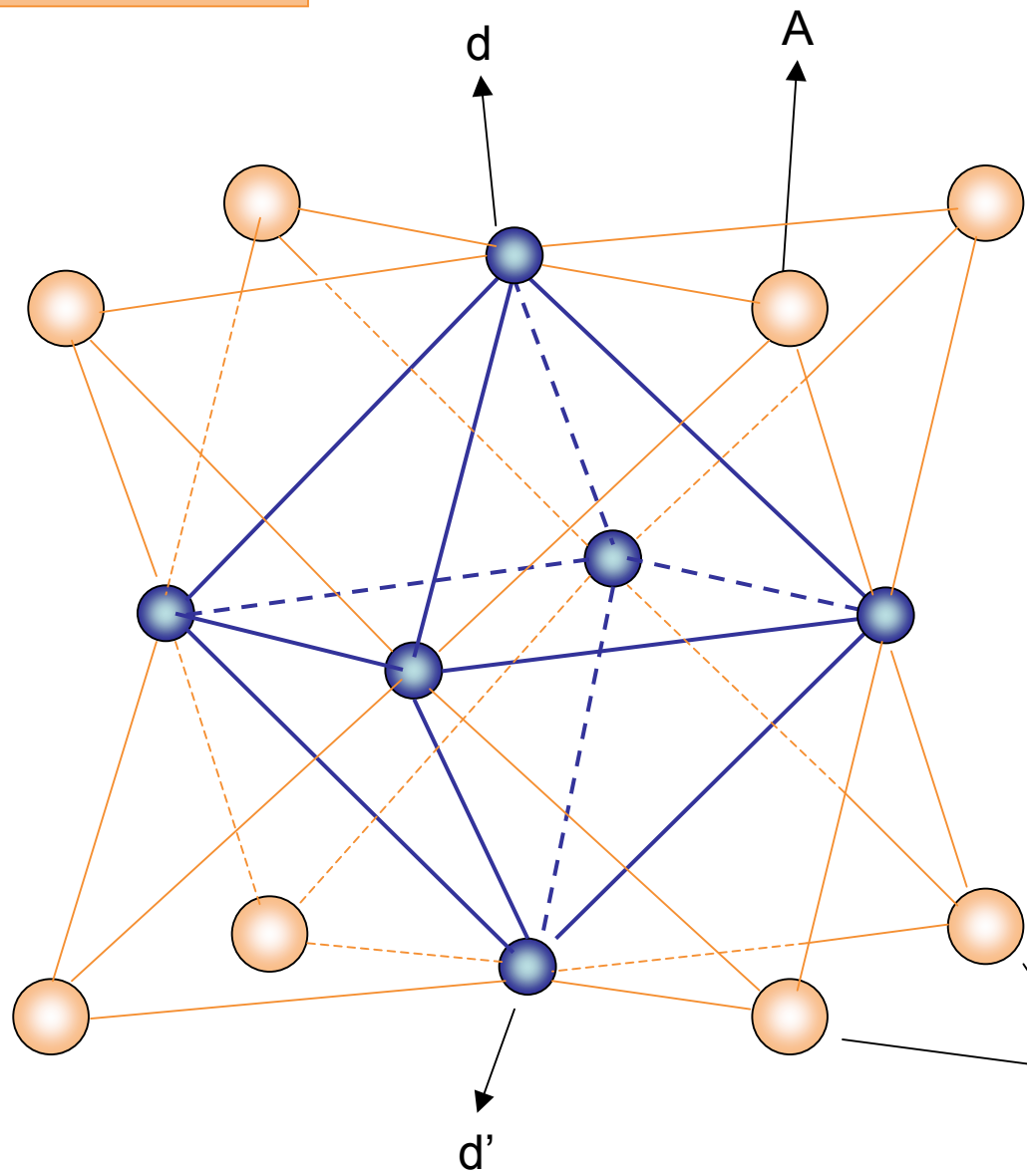
$$R_{d'e} = 1.22R_{dd}$$



$$\begin{aligned} & 12(e^2/R_{ee}) \\ & + 12e^2/(1.414R_{ee}) \\ & + 4e^2/(1.732/R_{ee}) \end{aligned}$$

$$R_{ee} = R_{dd}$$

24 dde faces



6D(2-)/OSC

$$R_B = 52.9 \text{ pm}$$

$$R_{dd} = 1 (=1.414R_B)$$

$$R_{d\text{-cross}} = 1.414$$

$$R_{d\text{-CM}} = 1/1.732$$

$$R_{dA} = 1/1.414(=R_B)$$

$$R_{A\text{-CM}} = 1/2.449$$

$$R_{ed'} = 1.732/1.414$$

Electron-Center

Langevin Equation for 6D(2-)/OSC

$$12m_d \frac{d^2 R_{dd}(t)}{dt^2} = -BA \frac{29.3}{[R_{dd}(t)]^2} - 24 \frac{\partial V_s(R_{dd}(t); 1,1)}{\partial R_{dd}(t)} + f(t)$$

12 (d-d) edges

Main condensation force of 6D²⁻

Constraint by 24 "dde" faces

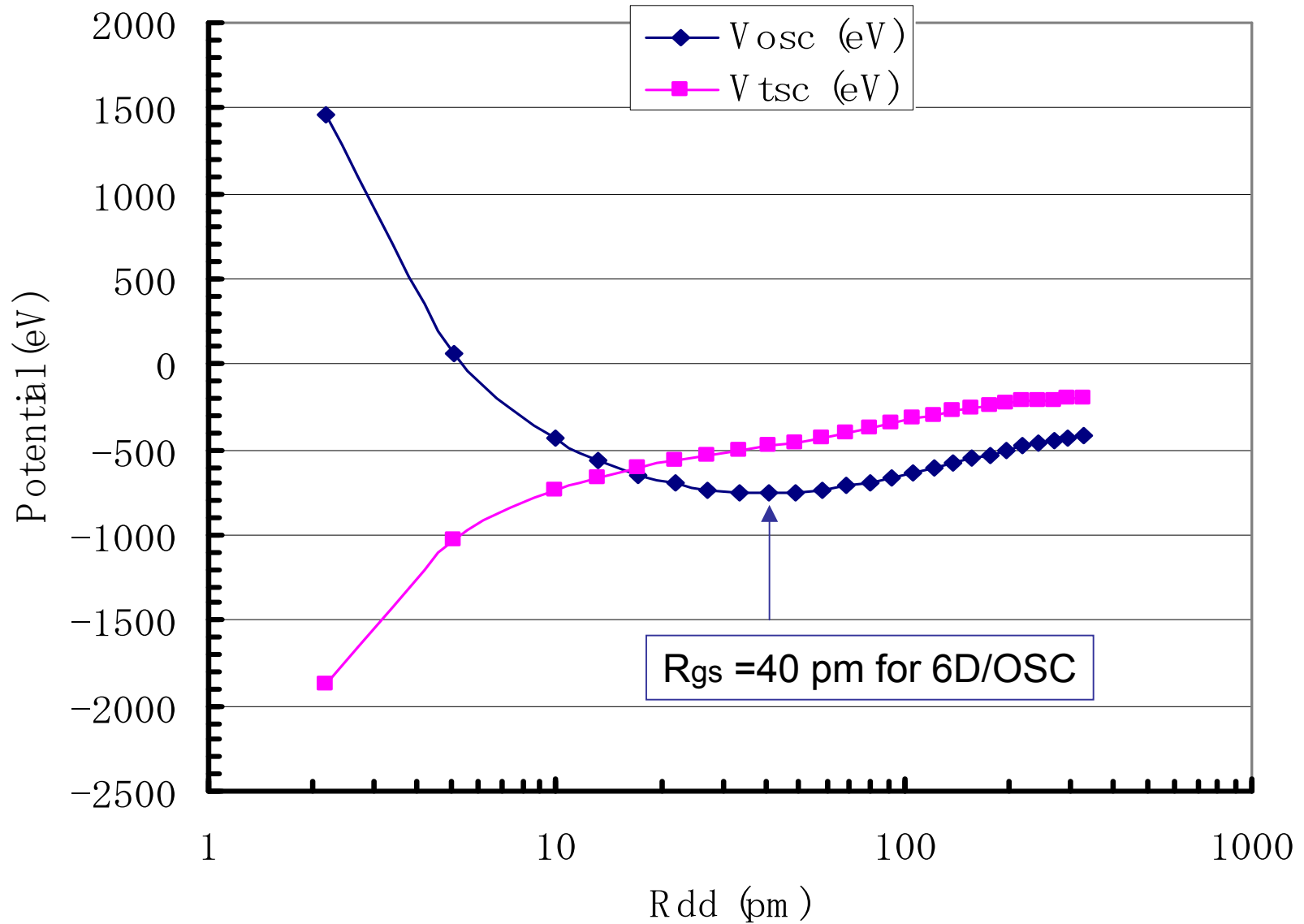
$$f(t) = \left[-\frac{\partial \Delta E_c(R_{dd})}{\partial R_{dd}} \right] \text{mod} [X^2(R'_{dd}; R_{dd}(t))]$$

Ratio=29.3/24=1.22
CF: 1.98 for 4D/TSC

Marginal!

Condensation might converge at $R_{dd}=10\text{pm}$!
This small negative entity nuclear-reacts with host metal nucleus by attraction?

Main Trapping Potential of 4D/TSC and 6D/OSC



6. HMEQPET Method for Fusion Rate Quantification

- The role and merit of HMEQPET (heavy mass electronic quasi-particle expansion theory) method for approximating time-dependent TSC trapping potential and relating to the estimation of time-dependent Coulomb barrier penetration probabilities of 4d cluster is explained. HMEQPET provides a practical method for calculating time-dependent (hence time-averaged) fusion rate under TSC condensation, based on the Fermi's first golden rule.

A Slice of Time-Dependent TSC Trapping Potential, which keeps balancing back to the Platonic Symmetric State

V_{tsc} (keV) vs. R' at $R_{\text{dd}}(t)=25$ fm using $V_{\text{s}}(2,2)$

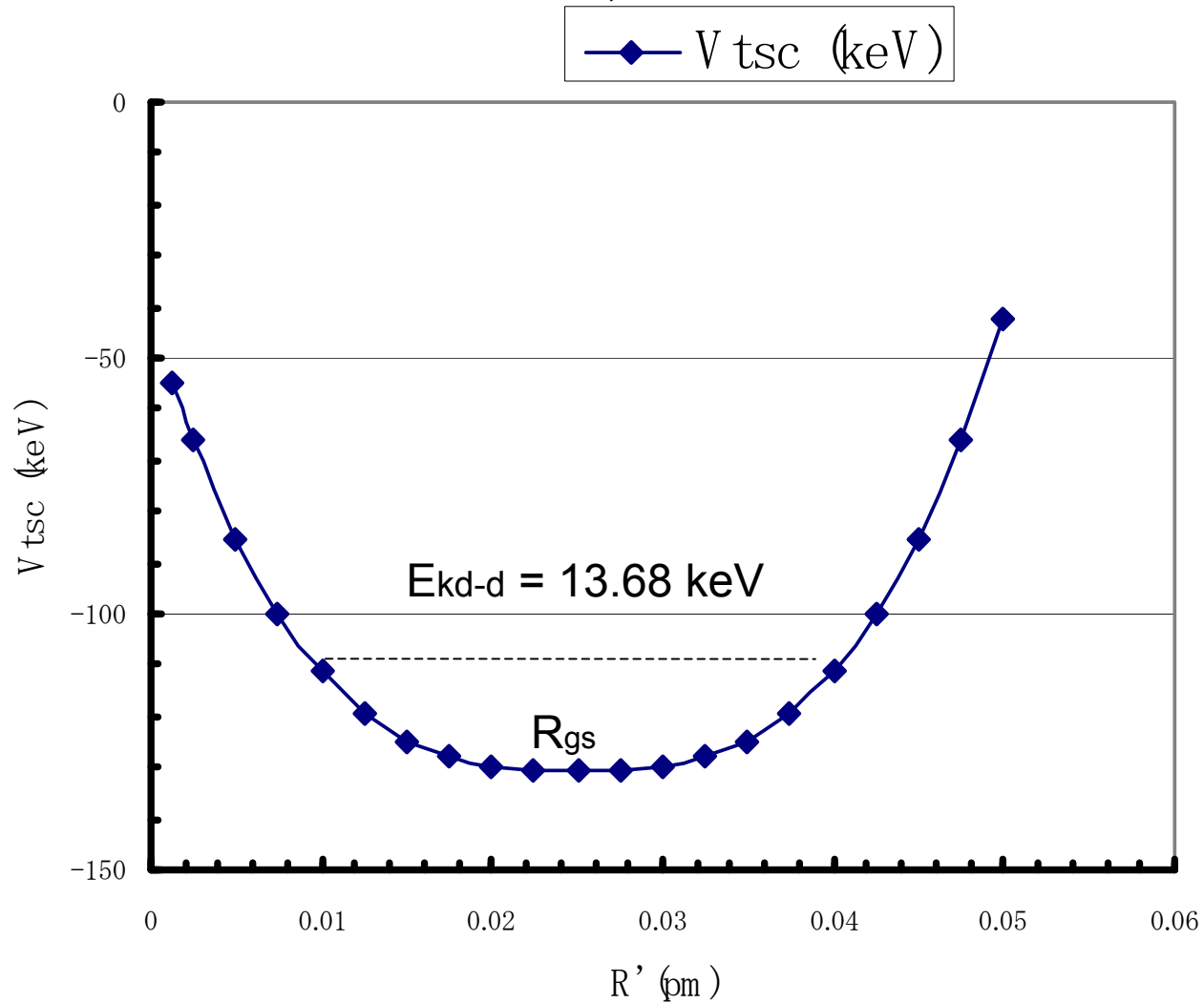


Table-3: Calculated HMEQPET potentials and their parameters,
Egs-Vs-min gives mean relative kinetic energy of trapped d-d pair

Molecule	b0 (pm)	Rmin (pm)	Vs-min (keV)	Ed-d (keV)	Rgs (pm)	Egs (keV)
D₂	22	70	-0.03782	0.00268	76.69	-0.03514
dde*(2,2)	4.5	19.3	-0.1804	0.01013	21.82	-0.17027
dde*(5,2)	1.9	7.6	-0.4509	0.0208	8.72	-0.43007
dde*(10,2)	0.90	3.8	-0.9019	0.0418	4.36	-0.86012
dde*(20,2)	0.45	1.9	-1.8039	0.0837	2.18	-1.7202
dde*(50,2)	0.18	0.76	-4.5097	0.2094	0.873	-4.3003
dde*(100,2)	0.09	0.38	-9.0194	0.4196	0.436	-8.5998
dde*(200,2)	0.045	0.19	-18.039	0.843	0.218	-17.196
dde*(500,2)	0.018	0.076	-45.097	2.135	0.0873	-42.968
dde*(1000,2)	0.009	0.038	-90.194	4.336	0.0436	-85.858
dde*(2000,2)	0.0045	0.019	-180.39	8.984	0.0218	-171.406

Due to HUP, relative d-d kinetic energy should go up finally to about 10keV.

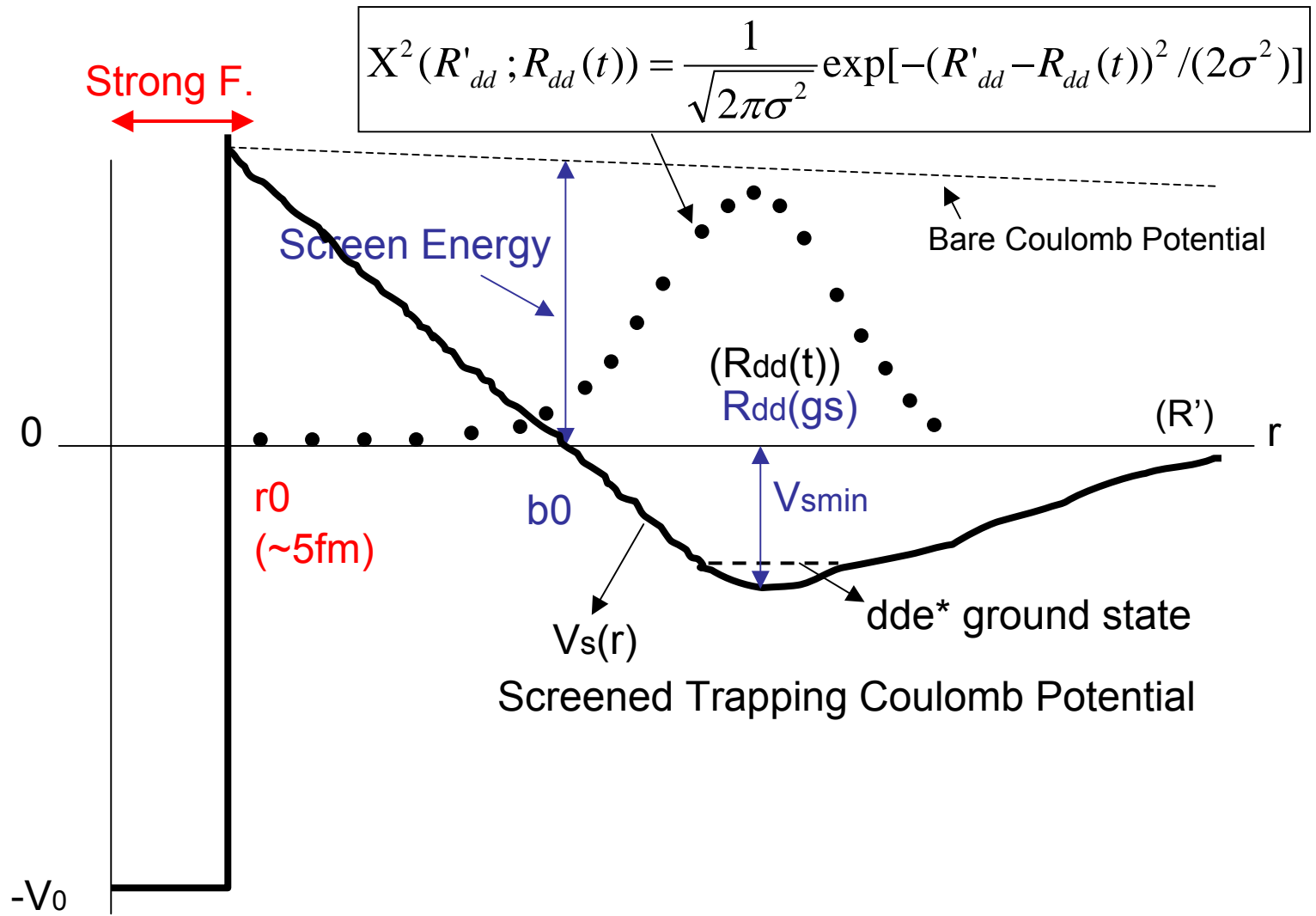
HMEQPET Method

- Time-dependent (R_{dd} -dependent) d-d trapping potential of TSC can be approximated by $V_s(R;m,2)$ potential for $dde^*(m,2)$ EQPET molecule.
- Heavy Mass Cooper Pair Concept: $e^*(m,2)$
- We got empirical relations:

$$b_0(m,2) = 0.206R_{gs}(m,2)$$

$$m = 9000 / b_0(m,2)$$

Adiabatic Potential for Molecule dde^* and its ground state squared wave function



Barrier Factor by Heavy Mass EQPET (HMEQPET)

$$b_0(m, Z) = 0.206 R_{gs}(m, Z)$$

$$m = 9000 / b_0(m, Z)$$

$$\Gamma_{dd}(m, Z) = 0.218 \sqrt{\mu} \int_{r_0}^{b_0(m, Z)} \sqrt{V_s(R_{dd}; m, Z) - E_d} dR_{dd}$$

$$P_{nd}(m, Z) = \exp(-n \Gamma_{dd}(m, Z))$$

Instead of biasing V_s potential with $-V_{s-\min}$,
we regard that

Γ_{dd} for $R_{gs}=b_0$ is approximate solution.

Table-2: Calculated time-dependent (equivalently R_{dd} dependent) barrier factors of 4D/TSC condensation motion

Elapsed Time (fs)	R_{dd} (pm)	P_{2d} : 2D barrier facotor	P_{4d} : 4D barrier factor
0	74.1 (D_2 molecule)	1.00E-85	1.00E-170
1.259	21.8 (dde*(2,2); Cooper pair)	1.30E-46	1.69E-92
1.342	10.3	2.16E-32	4.67E-64
1.3805	4.12	9.38E-21	8.79E-41
1.3920	2.06	6.89E-15	4.75E-29
1.3970	1.03	9.69E-11	9.40E-21
1.39805	0.805 (muon-dd molecule)	1.00E-9	1.00E-18
1.39960	0.412	9.40E-7	2.16E-13
1.40027	0.206	3.35E-5	1.12E-9
1.40047	0.103	1.43E-3	2.05E-6
1.40062	0.0412	1.05E-2	1.12E-4
1.40070	0.0206 (TSC-min)	4.44E-2	1.98E-3

7.1. Fusion Rate for Steady dde* molecule

- **<Fusion Rate per pair> = $T_n | \Psi(r_0) |^2$**

$$T_n = (4\pi/h) \langle \Psi_f | W(R) | \Psi_i \rangle / \langle \Psi_f | \Psi_i \rangle$$

$W(R)$: imaginary part of nuclear optical potential

$$H_{int} = U(R) = V(R) + iW(R)$$

$| \Psi(r_0) |^2$: (Coulomb barrier penetration probability at $R=r_0$)

R : d-d distance

Fusion Rates of Steady State dde* Molecules:

$$\lambda_{nd} = \frac{2}{\hbar} \langle W \rangle P_{nd}(r_0) = 3.04 \times 10^{21} P_{nd}(r_0) \langle W \rangle$$

Regarding b_0 as R_{gs} , we get $P_{nd}(r_0)$ values.

Here r_0 is 5 fm.

Molecule	R _{dd} =R _{gs} (pm)	P _{nd} (r ₀) ; Barrier- Factor	<W> (MeV)	λ _{2d} (f/s)	λ _{4d} (f/s)
D ₂	74.1	1.0E-85	0.008	2.4E-66	
dde*(2,2)	21.8	1.3E-46	0.008	3.2E-27	
μ dd	0.805	1.0E-9	0.008	2.4E+10	
4D/TSC-min	0.021	1.9E-3	62		3.7E+20

4D/TSC-min exists within $\Delta t = 2 \times 10^{-20}$ s at final stage of condensation:

Decay of TSC: $\exp(-\lambda_{4d} \Delta t) = \exp(-7.6) = 0.0006 \rightarrow$ **4D fusion by 100% per TSC Generation. 4D Fusion Rate (1/s) becomes 4D/TSC Production Rate (1/s).**

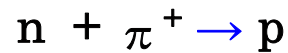
<W> value Estimation

- Using $T_n \sim (\text{PEF})^5$ in S-value analysis:

Cluster	<W> (MeV)
DD	0.008
DT	0.115
3D	1.93
4D	62.0

Scaling of PEF (Pion Exchange Force) for Nuclear Fusion by Strong Interaction

Two Body Interaction: $PEF = 1$



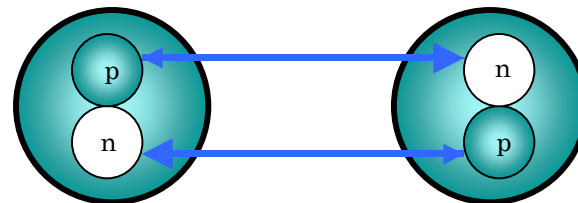
(udd) (ud*) (uud) : u ; up quark



(uud) (u*d) (udd) : u* ; anti-up quark

: d* ; anti-down quark

For D + D Fusion; $PEF = 2$



One Pion Exchange Potential and PEF

One Pion Exchange Potential (Hamada-Johnston Potential)

$$V_{OPEP}(x) = v_0 \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2) \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) S_{12} \right\} \frac{\exp(-x)}{x}$$

$$x = \frac{m_\pi c}{\hbar} r = \frac{r}{1.43} [fm] \quad \text{Yukawa Potential} \quad Y(x) = \frac{\exp(-x)}{x}$$

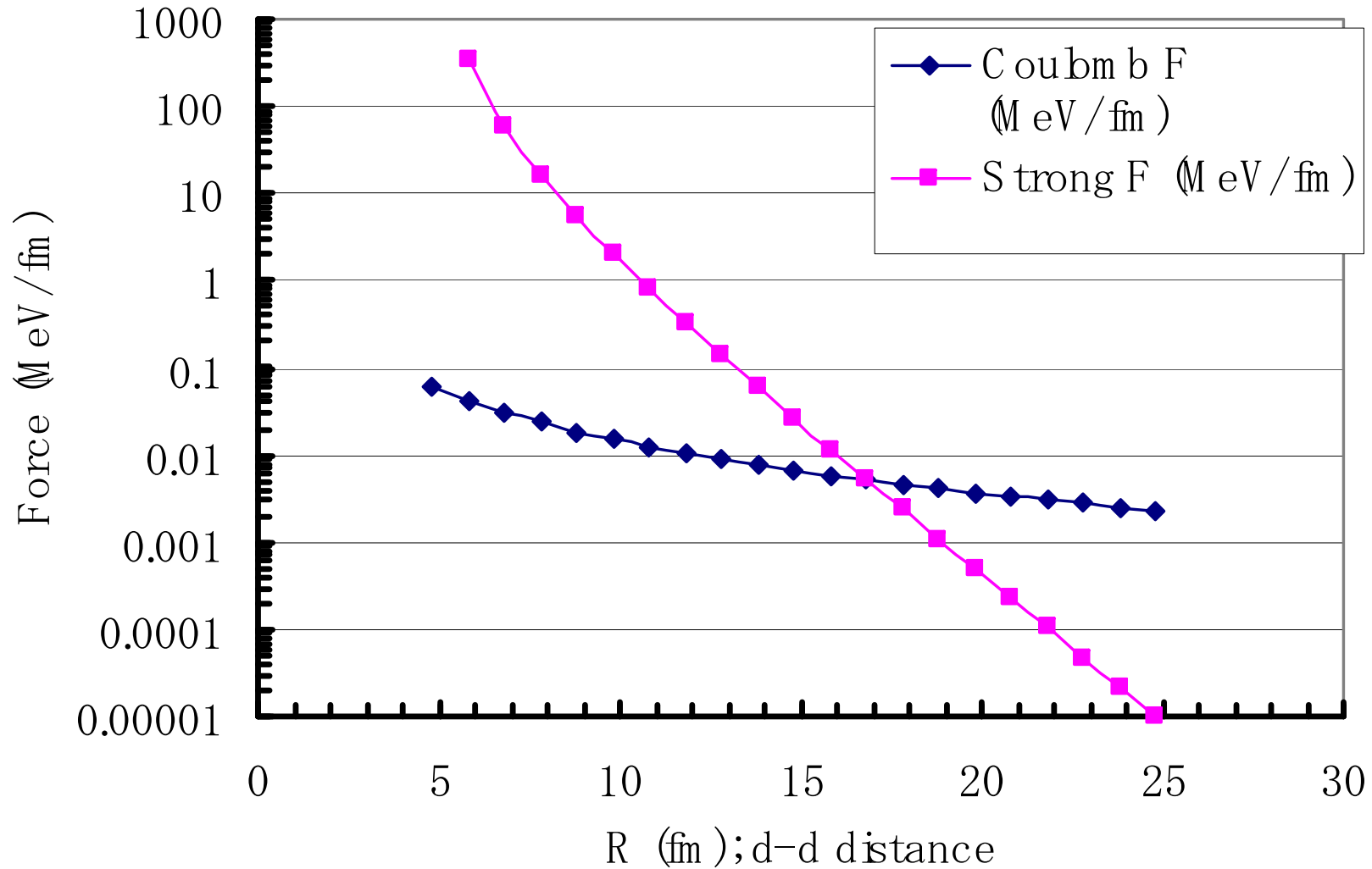
$$v_0 = \frac{1}{3} \frac{f^2 m_\pi c^2}{\hbar c} = 3.65 [MeV] \quad S_{12} = 3 \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$\langle \text{OnePEF} \rangle = - \frac{\partial \langle V_{OPEP}(x) \rangle_{\tau, \sigma}}{\partial r} = - \frac{1}{1.43} \frac{\partial \langle V_{OPEP}(x) \rangle_{\tau, \sigma}}{\partial x}$$

D-D Fusion: Strong Force vs. Coulomb Force

Yukawa Force Attraction vs. Coulomb Force Repulsion

PEF=2 for d-d interaction



Estimation of S-value

- Scaling by PEF-values:

$$U(r) = V(r) + iW(r)$$

$$W(r) \sim W_0 \delta(r - r_0)$$

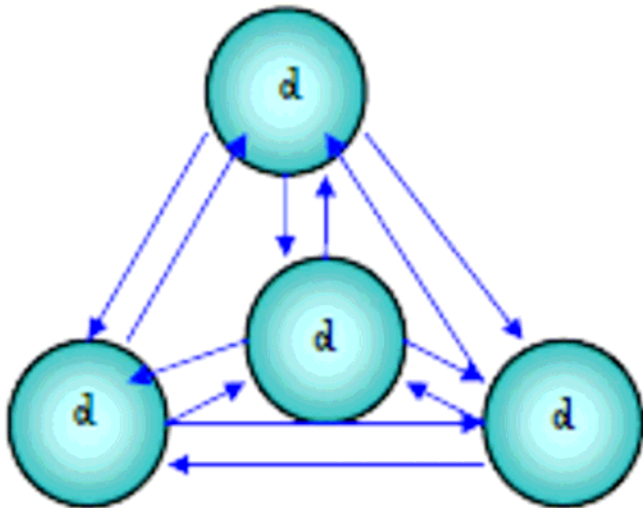
- PEF reflects size of contact sticking surface for fusion reaction by charged pion exchange:

$$S_n(0) \sim T_n^2 \sim (\text{PEF})^N$$

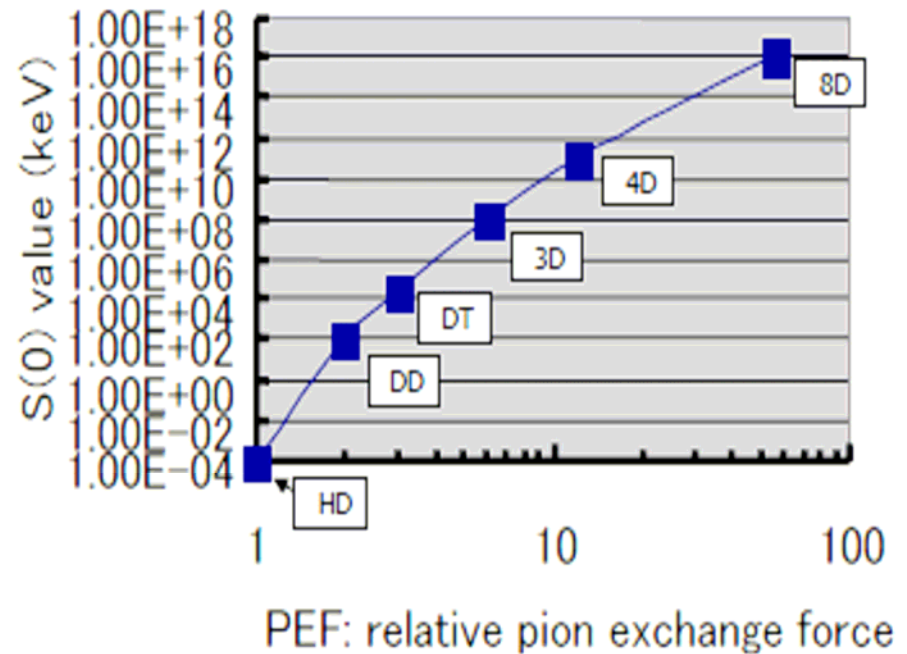
- $S_{dd}=1.1E2$ keVb, $S_{dt} = 2E4$ keVb, with $\text{PEF}_{dd} = 2$, $\text{PEF}_{dt}=3$, $\text{PEF}_{4d}=12$
- N is roughly 11.4 to give $S_{4d} = 1E11$ keVb

Extrapolation of $S(0)$ to multi-body D-fusion

4D Fusion; PEF = 12



$S(0)(\text{keVb})$ vs. PEF



7.2. 4D Fusion and ^4He Production Rate by TSC (Time-Dependent Fusion Rate)

- t_c : Condensation Time of TSC (**1.4007 fs**)
- η_{4d} : 4D Fusion Yield per TSC

$$\eta_{4d} = 1 - \exp\left(-\int_0^{t_c} \lambda_{4d}(t) dt\right)$$

$$\lambda_{4d}(t) = 3.04 \times 10^{21} \langle W \rangle P_{4d}(r_0; R_{dd}(t)) = 1.88 \times 10^{23} P_{4d}(r_0; R_{dd}(t))$$

$$\int_0^{t_c} \lambda_{4d}(t) dt = 1.88 \times 10^{23} \int_0^{t_c} P_{4d}(r_0; R_{dd}(t)) dt \quad \int_0^{t_c} P_{4d}(r_0; R_{dd}(t)) dt = 2.31 \times 10^{-22}$$

$$Y_{4d} = Q_{tsc} \eta_{4d}$$

Macroscopic 4D Fusion Production Rate



$$\eta_{4d} \cong 1.0$$

$$Y_{4d} \approx Q_{tsc}$$

Q_{tsc} : TSC Generation Rate

8. Decay-Channel of ${}^8\text{Be}$ for S-Wave

$$4\text{D} \rightarrow {}^8\text{Be}(0^+; 47.6 \text{ MeV}) :$$

$${}^8\text{Be}(0^+) \rightarrow {}^4\text{He}(0^+) + {}^4\text{He}(0^+) + 91.86 \text{ keV}$$

Others (below) are forbidden by Spin-Parity Conservation (Odd parity)

- ${}^3\text{He} (1/2^+) + {}^5\text{He}(3/2^-)(n+{}^4\text{He}) - 11.13 \text{ MeV}$
- $t (1/2^+) + {}^5\text{Li}(3/2^-)(p+{}^4\text{He}) - 21.68 \text{ MeV}$
- $p (1/2^+) + {}^7\text{Li} (3/2^-) - 17.26 \text{ MeV}$
- $n(1/2^+) + {}^7\text{Be}(3/2^-) - 18.90 \text{ MeV}$

Initial Spin Combination of 4d/TSC; **S-Wave**: Only 4d(0+) makes fusion

Entrance Channel

- $\uparrow\uparrow\uparrow\uparrow$; $J^\pi=4+$, $T=0$
(2 cases)
 - $\uparrow\uparrow\uparrow\downarrow$; $J^\pi= 2+$, $T=0$
(8 cases)
 - $\uparrow\downarrow\uparrow\downarrow$; $J^\pi= 0+$, $T=0$
(6 cases)
 - Other cases: $J^\pi= 3-$, $1-$
for total 4d spin
- Deuteron : $J^\pi= 1+$

Out-going Channel

- ${}^8\text{Be}^*(4+ : 47.6\text{MeV})$ to $2^4\text{He}(0+ : \text{gs})$; **forbidden**
- ${}^8\text{Be}(2+ : 47.6\text{MeV})$ to ${}^4\text{He}(0+ : 20.21\text{ MeV}) + {}^4\text{He}(2+ : 27.42\text{MeV}) - 0.03\text{MeV}$; **forbidden**
- ${}^8\text{Be}(0+ : 47.6\text{MeV})$ to $2^4\text{He}(0+ : \text{gs}) + 47.6\text{MeV}$ is **allowed. (37.5%?)**

Discussion on 2D Fusion Rate

- $\tau_{(2,2)}$: Life Time of $dde^*(2,2)$
- η_{2d} : Fusion Yield per dd pair

$$\eta_{2d} = 1 - \exp(-\lambda_{2d(2,2)} \tau_{(2,2)})$$

$$\eta_{2d} \approx \lambda_{2d(2,2)} \tau_{(2,2)}$$

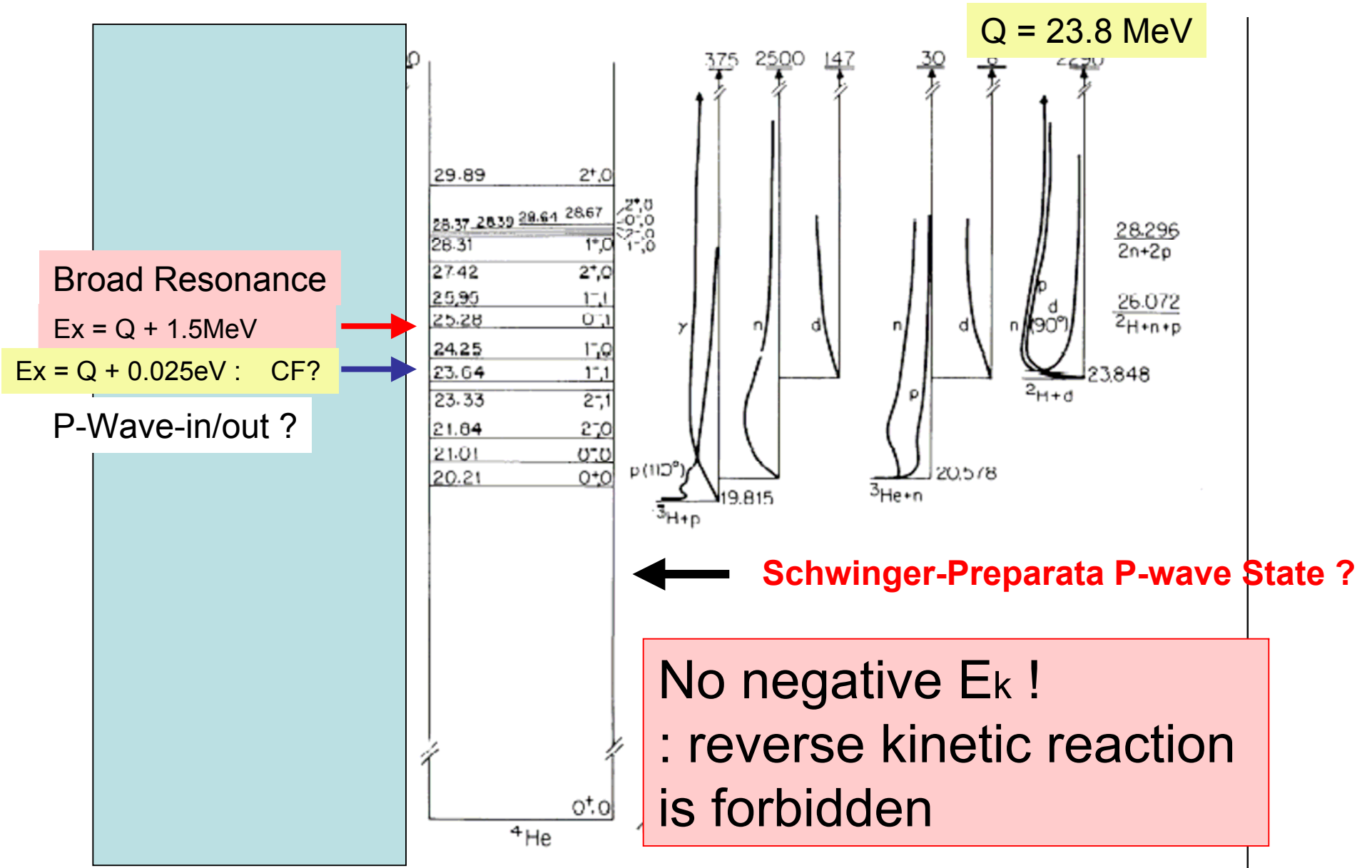
$$Y_{2d} = Q_{dde^*(2,2)} \eta_{2d}$$

a) If $\tau_{(2,2)}$ is 10^4 s (as X. Z. Li asserts), $\eta_{2d} = 3.1 \times 10^{-23}$

Assuming $Q_{dde^*} = 10^{22}$ (1/s/cc), $Y_{2d} = 0.3$ (f/s/cc)

b) 2D Fusion rate may be larger, when TSC under condensation would break up to two dde^* diminished size molecules. We need Monte-Carlo Langevin cal.

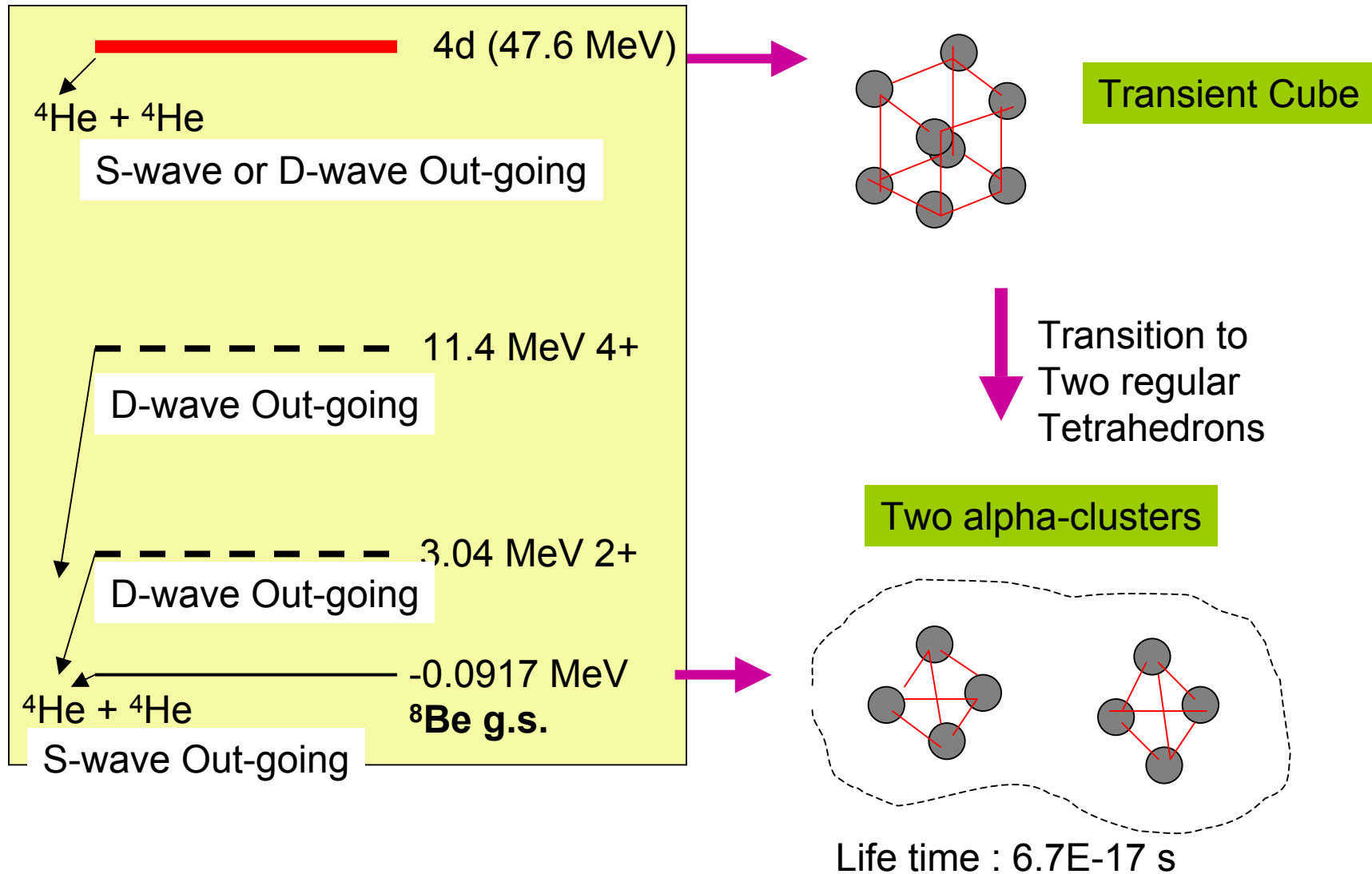
$$d + d + E_k = {}^4\text{He}^*(E_x) = {}^4\text{He}^*(Q + E_k)$$



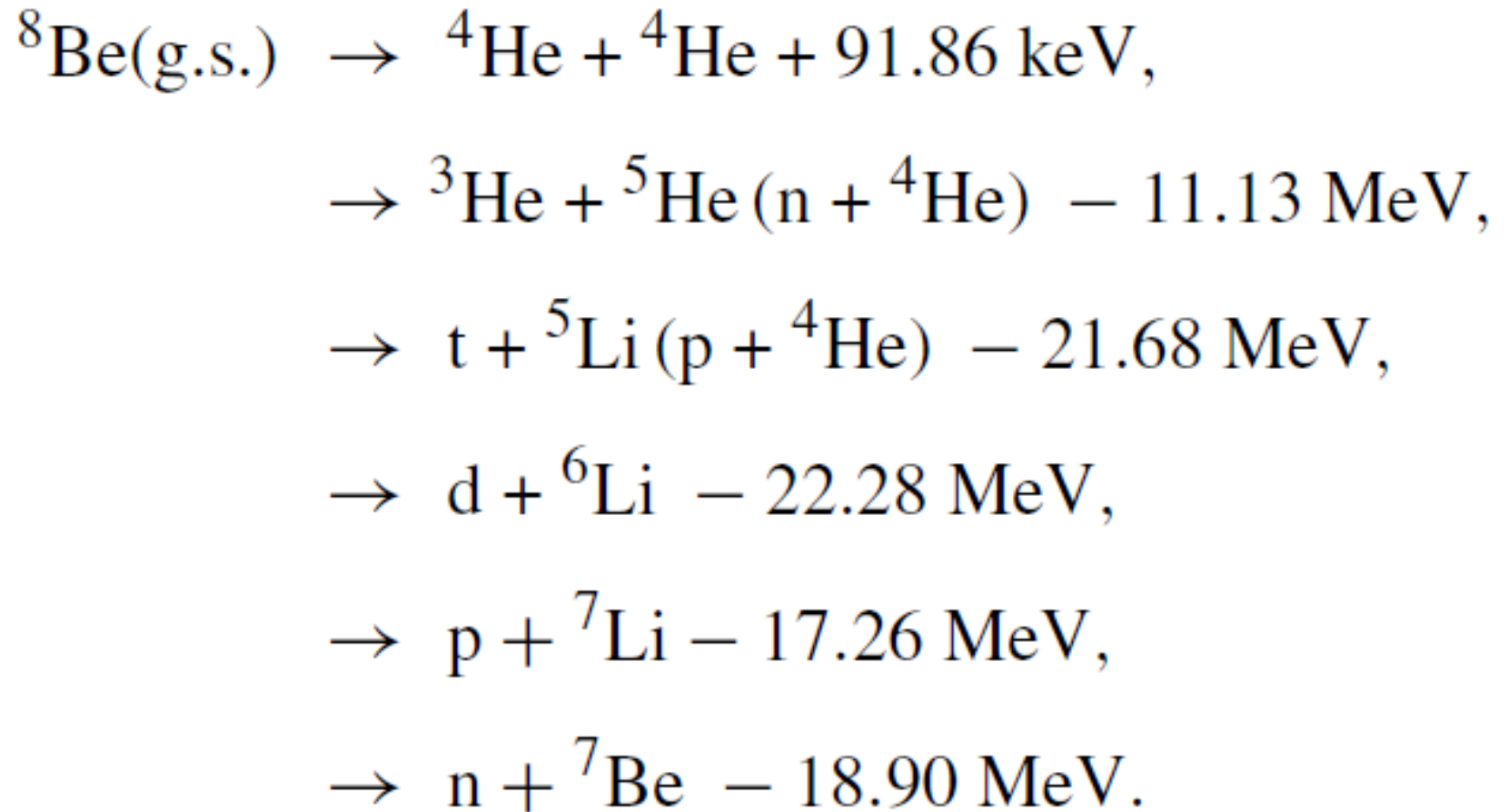
D. Tilley, H. Weller, G. Hale: Nucl. Phys. A541,
(1992)1-104 (Page 15)

Nestled among a series of four negative-parity, $T = 1$ levels in the range $E_x = 23 - 26$ MeV (which were at least 3 MeV higher in (1973FI04)) is a new $1^-, T = 0$ level at $E_x = 24.25$ MeV that has important effects on the $d + d$ reactions at low energies. Isospin mixing between this state and the ${}^3P_1, T = 1$ level at $E_x = 23.64$ MeV causes significant differences in the p-wave part of the $d + d$ reactions, as have been observed in muon-catalyzed (1984BA1W) and polarized (1981AD07) $d + d$ fusion experiments. The $1^-, T = 0$ level was seen by (1981GR16) in their ${}^2\text{H}(d, p){}^3\text{H}$ analyzing-power data, but no evidence of their proposed 4^+ level at $E_x = 24.6$ MeV was found by (1989HA2A) in fitting their measurements.

$4D \rightarrow {}^4\text{He} + {}^4\text{He} + 47.6\text{MeV}$ (Final State Interaction)



Final State Interaction of ${}^8\text{Be}^*$ ($E_x=47.6\text{MeV}$)



Summary-1

- **Platonic Symmetric Arrangement** realizes Energy-Minimum State of Many-Body System.
- PA appears in D-atom, D_2 , D_2^+ , D_3^+ , and 4D/TSC.
- PA appears in CMNS of 4D/TSC for Coulombic Interaction and Strong Interaction.
- Dynamic PA is of key for 4D Cluster Fusion.
- Good solution by Molecular Dynamics with Langevin Eq., for Platonic Systems as, D, D_2 , D_3^+ and 4D(H)/TSC.
- About 100% 4D-Fusion per TSC generation!

Summary 2

- Only 4D(H)/TSC can condense ultimately to 10-20 fm radius size.
- Bosonized $e(1/2)+e(-1/2)$ coupling for dde system makes D_2 type faces of TSC to help ultimate condensation.
- $6D^2$ -/OSC converges its condensation at about $R_{dd}=40$ pm, but closer d-d distance in transient.
- Single $\langle e^- \rangle$ -center states for dde (D_2^+) faces of OSC enhances constraint (friction) for condensation.

9. Conclusions

- One dimensional **Langevin equations** for treating dynamic motion of D-clusters under Platonic symmetry were formulated.
- **Only 4D/TSC makes ultimate condensation** to form 10-20 fm size charge-neutral entity, among D_2 , D_3^+ , 4D/TSC and $6D^2$ -/OSC.
- **Almost 100% (Smaller by spin) 4D fusion** with two ^4He ashes may take place by one 4D/TSC generation in metal-D system dynamics.