Theory of Bose-Einstein Condensation Nuclear Fusion*

Yeong E. Kim
Purdue Nuclear and Many-Body Theory Group
Department of Physics, Purdue University
West Lafayette, IN 47907

Presented at
Vice Chancellor for Research Seminar Series
University of Missouri, Columbia, Missouri
29 May 2009

***Based on a paper published on-line**
Naturwissenschaften DOI 10.1007/s00114-009-0537-6
(14 May 2009)

I. Introduction and Background

II. Theory

III. Predictions and Comparisons

IV. Proposed Experimental Tests

V. Conclusion

- (D+D) fusion in free space ($E \geq 10$ keV):

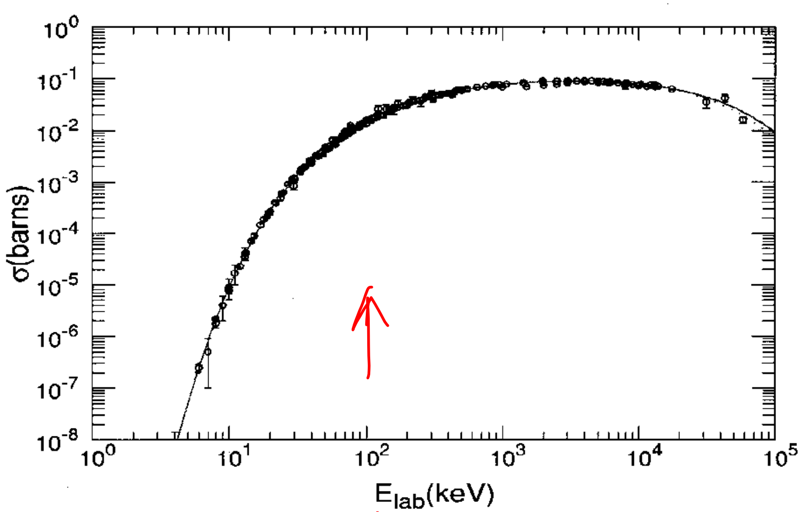
$\{1\}$ $D + D \longrightarrow p + T + 4.03$ MeV

$\{2\}$ $D + D \longrightarrow n + {}^3He + 3.27$ MeV

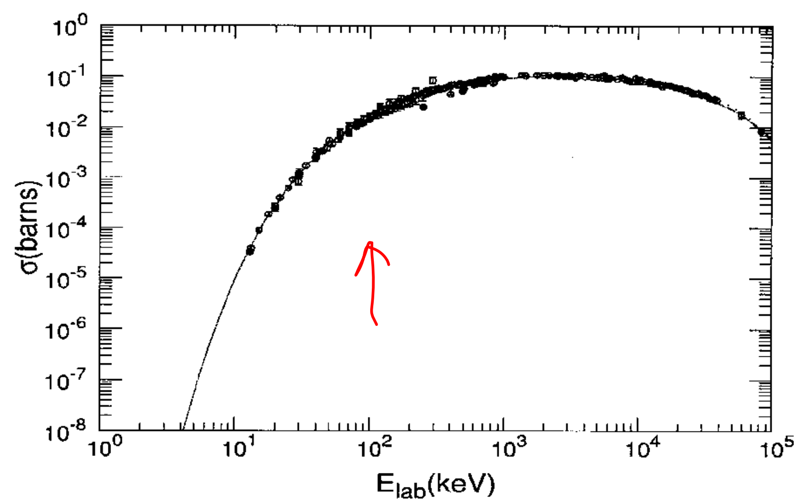
$\{3\}$ $D + D \longrightarrow {}^4He + \gamma + 23.8$ MeV

Branching ratios of the measured cross-sections: $(\sigma\{1\}, \sigma\{2\}, \sigma\{3\}) \approx (0.5,\ 0.5,\ \sim 10^{-6})$

Reaction {1}



Reaction {2}



For $E_{lab} < 100$ keV, the fit is made with $\sigma(E) = \dfrac{\mathbf{S}}{\mathbf{E}}\, \mathbf{e}^{-\sqrt{E_G}/\sqrt{E}}$

Experimental Observations

(D+D) fusion in metal (E ≤ 10 eV) (m represents a host metal lattice or metal particle) :

{4} $D(m) + D(m) \rightarrow p(m) + T(m) + 4.03$ MeV(m)

{5} $D(m) + D(m) \rightarrow n(m) + {}^3He(m) + 3.27$ MeV(m)

{6} $D(m) + D(m) \rightarrow {}^4He(m) + 23.8$ MeV (m)

Fusion rate R{6} for {6} is much greater than rates R{4} and R{5}

- **Initial Claim: Radiationless fusion reaction (Electrolysis Exp.)**

  {6} $D(m) + D(m) \rightarrow {}^4He(m) + 23.8$ MeV (m)

- The above fusion reactions are not expected to be observable at room temperature

  (1) due to the DD Coulomb repulsion (Gamow factor), and

  (2) due to the violation of the momentum conservation in free space.

Estimates of the Gamow factor $T_G(E)$ for D + D fusion with electron screening energy U_e

$$T_G(E) = e^{-\sqrt{E_G}/\sqrt{E}}, \qquad E_G = \frac{(2\pi\alpha Z_1 Z_2)^2 \mu c^2}{2} \quad \text{(Gamow Energy)}$$

$$\mathrm{E} \rightarrow \mathrm{E} + \mathrm{U_e}, \; \mathrm{T_G}(E + U_e) = e^{-\sqrt{E_G}/\sqrt{E+U_e}}$$

$E+U_e$	$T_G(E + U_e)$	U_e	$r_{screening}$
1/40 eV	10^{-2760}	0	
14.4 eV	10^{-114}	14.4 eV	1 Å
43.4 eV	10^{-65}	43.4 eV	0.33 Å
~300 eV	10^{-25}	300 eV	
~600 eV	10^{-18}	600 eV	

- Values of Gamow Factor $T_G(E)$ extracted from experiments

$T_G(E)^{FP} \approx 10^{-20}$ (Fleischmann and Pons, excess heat, Pd cathode)
$T_G(E)^{Jones} \approx 10^{-30}$ (Jones, et al., neutron from $D(d,n)^3He$, Ti cathode)

(D+D) fusion in metal (E< 10 keV) (m represents a host metal lattice or metal particle) :

{4} D(m) + D(m) → p(m) + T(m) + 4.03 MeV(m)

{5} D(m) + D(m) → n(m) + ^{3}He(m) + 3.27 MeV(m)

{6} D(m) + D(m) → ^{4}He(m) + 23.8 MeV

Experimental Observations (as of 2008) (not complete)

From both electrolysis and gas loading experiments

[1] The Coulomb barrier between two deuterons are suppressed

[2] Excess heat production (the amount of exess heat indicates its nuclear origin)

[3] "Heat-after-death"

[4] ^{4}He production comensurate with excess heat production, no 23.8 MeV gamma ray

[5] Production of nuclear ashes with anomalous rates: R{4} << R {6} and R {5} << R{6}

[6] Detection of radiations

[7] Production of hot spots and micro-scale crators on metal surface

[8] Requirement of deuteron mobility (D/Pd > ~0.9, electric current, pressure gradient, etc.)

[9] Requirement of deuterium purity (H/D << 1)

[10] More tritium is produced than neutron R(T) >> R(n)

Based on a single physical concept, can we come up with a consistent physical theory which could explain all of the following nine experimental observations ?

Deuterons become mobile in metal when electric current or heating is applied !

→ We will explore a concept of "nuclear" Bose-Einstein Condensation for developing a consistent physical theory.

Experimental Observations (as of 2008) (Electrolysis and gas loading)

[1] The Coulomb barrier between two deuterons are suppressed

[2] Excess heat production (the amount of excess heat indicates its nuclear origin)

[3] "Heat-after-death"

[4] ^{4}He production commensurate with excess heat production, no 23.8 MeV γ ray

[5] Production of nuclear ashes with anomalous rates: R{4} << R {6} and R {5} << R{6}

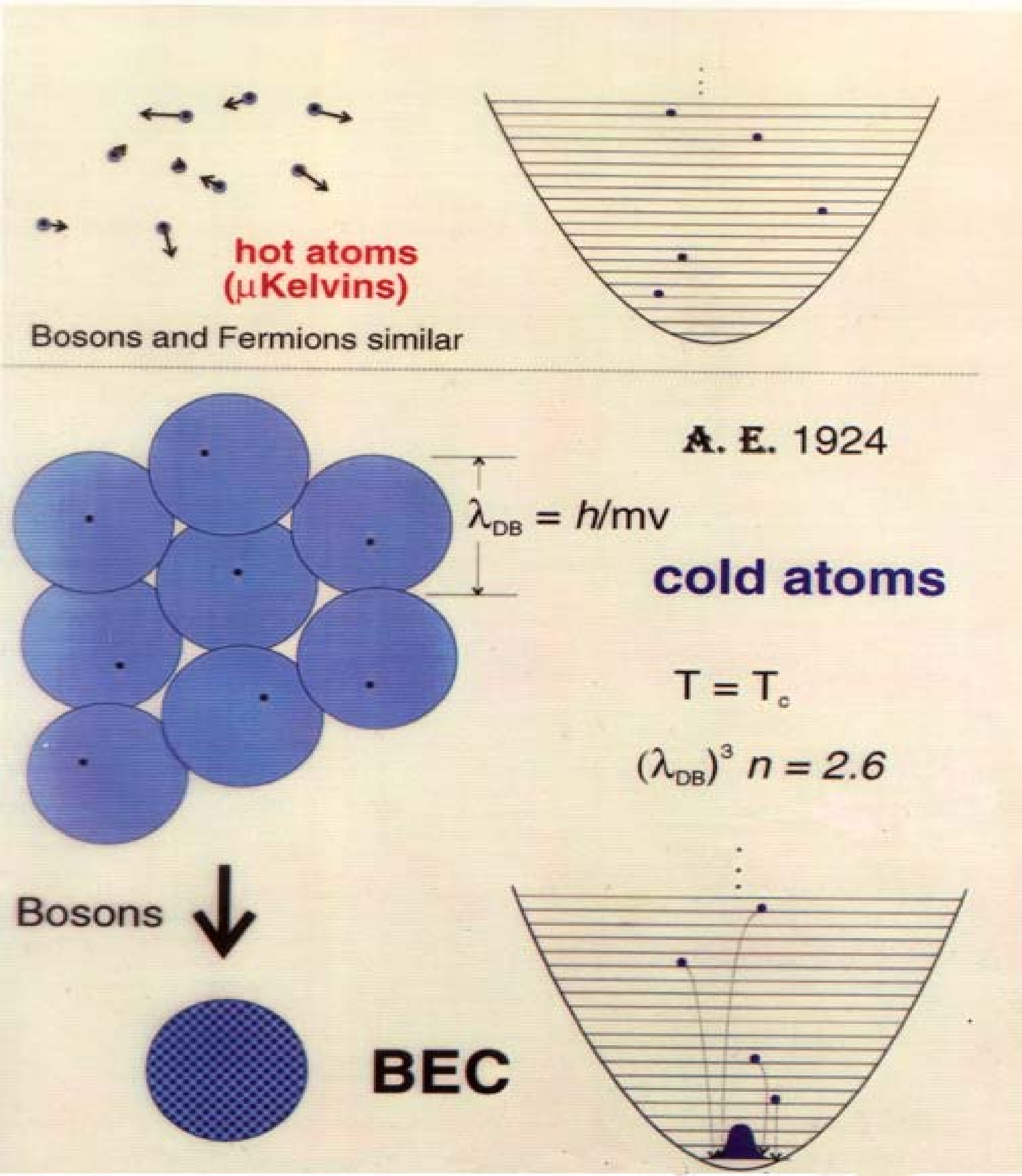
[6] Detection of radiations

[7] Production of hot spots and micro-scale craters on metal surface

[8] Requirement of deuteron mobility (D/Pd > ~0.9, electric current, pressure gradient, etc.)

[9] Requirement of deuterium purity (H/D << 1)

[10] More tritium is produced than neutron R(T) >> R(n)



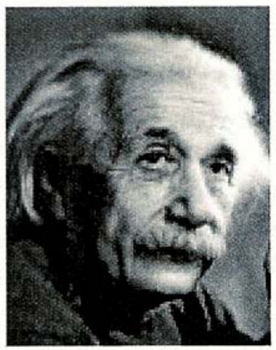
Requirement for Bose-Einstein Condensation (BEC):

$\lambda_{DB} > d$

where d is the average distance between neighboring two Bosons

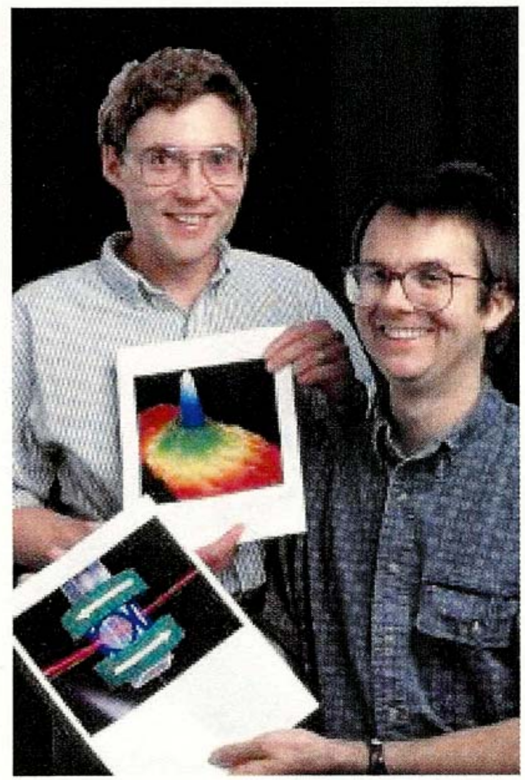
Bose-Einstein Condensation in a gas: a new form of matter at the coldest temperatures in the universe...

Predicted 1924... ...Created 1995

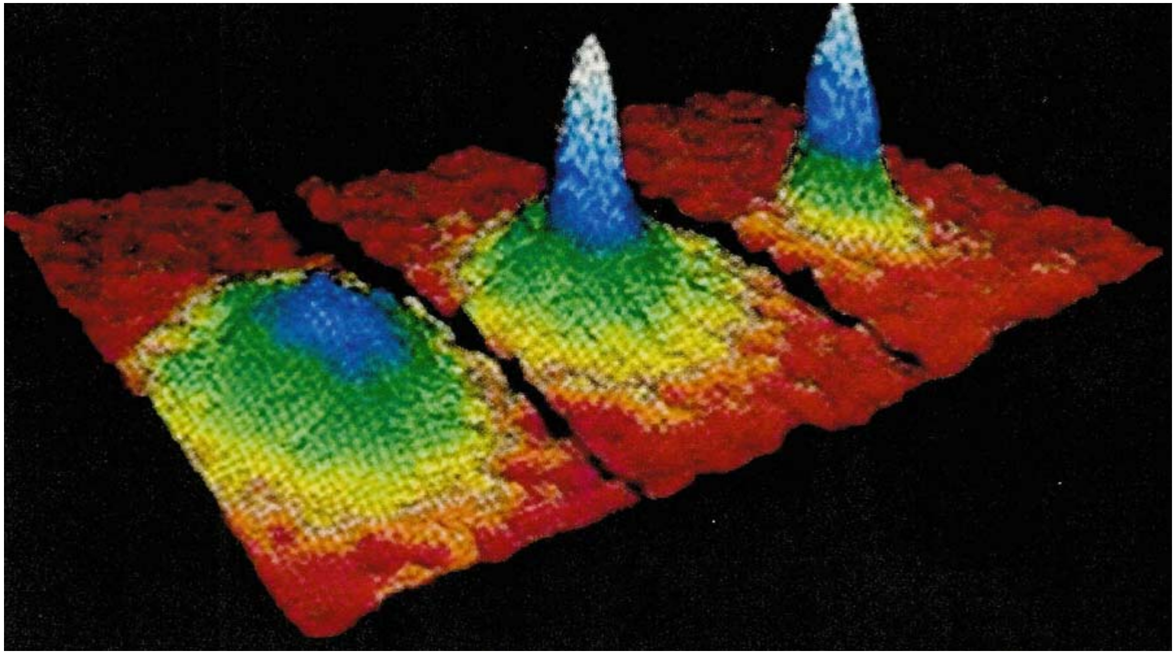


A. Einstein

S. Bose

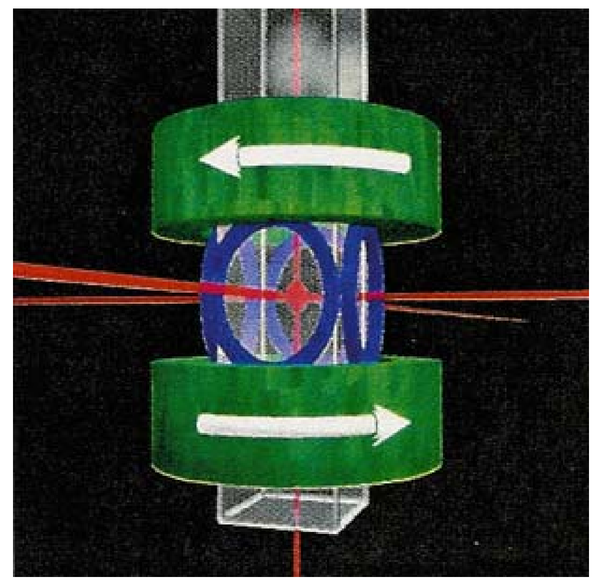


~ 400 nK ~ 200 nK ~ 50 nK



Plots of experimental data for 2-dimensional velocity distribution of Rb atoms in BEC as a function of temperatures

Schematic Picture of BEC Apparatus



Atomic BEC vs. Nuclear BEC

$$\lambda_{DB} > d \quad , \quad \lambda_{DB} = \frac{h}{m\upsilon}$$

Atomic BEC: $d \approx 7 \times 10^3$ Å $= 0.7$ μm (with $n_{Rb} = 2.6 \times 10^{12}/cm^3$)

$\upsilon_c \approx 0.6$ cm/sec near $T \approx 170$ n Kelvin

(BEC of ~ 2000 atoms out of ~ 2×10^4 atoms)

(1) Increase λ_{DB} by slowing down neutral atoms using laser and evaporation cooling

Nuclear BEC: $d \approx 2.5$ Å (with $n_D = 6.8 \times 10^{22}/cm^3$ in metal)

$\upsilon_c \approx 0.78 \times 10^5$ cm/sec ($\upsilon_{kT} \approx 1.6 \times 10^5$ cm/sec at T= 300 Kelvin)

(1) Increase λ_{DB} by slowing down charged deuterons using electromagnetic fields, and or

(2) Decrease d by compression using ultrahigh pressure device such as Diamond Anvil Cell (DAC)

Boson-Einstein Condensate (BEC) Mechanism
N-**Body Schroedinger Equation for the BEC State**

For simplicity, we assume an isotropic harmonic potential for the deuteron trap.

N-body Schroedinger equation for the system is

$$H\Psi = E\Psi \tag{1}$$

where Hamiltonian is given by

$$H = \frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \frac{1}{2}m\omega^2\sum_{i=1}^{N}r_i^2 + \sum_{i<j}\frac{e^2}{|r_i - r_j|} \tag{2}$$

where m is the rest mass of the nucleus.

In presence of electrons, we use the shielded Coulomb potential (Debye screening)

Equivalent Linear Two-Body (ELTB) Method

(Kim and Zubarev, Physical Review A **66**, 053602 (2002))

For the ground-state wave function Ψ, we use the following approximation

$$\Psi(\vec{r}, \ldots \vec{r}_N) \approx \tilde{\Psi}(\rho) = \frac{\Phi(\rho)}{\rho^{(3N-1)/2}} \tag{3}$$

where $\rho = \left[\sum_{i=1}^{N} r_i^2 \right]^{1/2}$

It has been shown that approximation (3) yields good results for the case of large N (Kim and Zubarev, J. Phys. B: At. Mol. Opt. Phys. **33**, 55 (2000))

By requiring that $\tilde{\Psi}$ must satisfy a variational principle $\delta \int \tilde{\Psi}^* H \tilde{\Psi} d\tau = 0$ with a subsidiary condition $\int \tilde{\Psi}^* \tilde{\Psi} d\tau = 1$, we obtain the following Schrödinger equation for the ground state wave function Φ(ρ)

$$[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + \frac{m}{2}\omega^2\rho^2 + \frac{\hbar^2}{2m}\frac{(3N-1)(3N-3)}{4\rho^2} + V(\rho)]\Phi = E\Phi \tag{4}$$

where $$V(\rho) = \frac{2N\Gamma(3N/2)}{3\sqrt{2\pi}\Gamma(3N/2-3/2)\rho} \tag{5}$$

Optical Theorem Formulation of Nuclear Fusion Reactions
(Kim, et al. Physical Review C 55, 801 (1997))

In order to parameterize the short-range nuclear force, we use the optical theorem formulation of nuclear fusion reactions. The total elastic nucleus-nucleus amplitude can be written as

$$f(\theta) = f^{c}(\theta) + \tilde{f}(\theta) \qquad (6)$$

where $f^{c}(\theta)$ is the Coulomb amplitude, and $\tilde{f}(\theta)$ can be expanded in partial waves

$$\tilde{f}(\theta) = \sum_{l} (2l+1) e^{2i\delta_l^c} f_l^{n(el)} P_l(\cos\theta) \qquad (7)$$

In Eq. (7), δ_l^c is the Coulomb phase shift, $f_l^{n(el)} = (S_l^n - 1)/2ik$, and s_l^n is the l-th partial wave S-matrix for the nuclear part.

For low energy, we can write (optical theorem)

$$\mathrm{Im}\, f_l^{n(el)} \approx \frac{k}{4\pi}\sigma_l^r \qquad (8)$$

where σ_l^r is the partial wave reaction cross section.

In terms of the partial wave t-matrix, the elastic scattering amplitude, $f_l^{n(el)}$ can be written as

$$f_l^{n(el)} = -\frac{2\mu}{\hbar^2 k^2} < \psi_l^c | t_l | \psi_l^c > \qquad (9)$$

where ψ_l^c is the Coulomb wave function.

Parameterization of the Short-Range Nuclear Force

For the dominant contribution of only *s*-wave, we have

$$\mathrm{Im}\, f_0^{n(el)} \approx \frac{k}{4\pi}\sigma^r \tag{10}$$

and

$$f_0^{n(el)} = -\frac{2\mu}{\hbar^2 k^2}\left\langle \psi_0^c \middle| t_0 \middle| \psi_0^c \right\rangle \tag{11}$$

Where σ^r is conventionally parameterized as

$$\sigma^r = \frac{S}{E} e^{-2\pi\eta} \tag{12}$$

$\eta = \frac{1}{2kr_B}, r_B = \frac{\hbar^2}{2\mu e^2}, \mu = m/2$, $e^{-2\pi\eta}$ is the "Gamow" factor,

and S is the S- factor for the nuclear fusion reaction between two nuclei.

From the above relations, Eqs. (10), (11), and (12), we have

$$\frac{k}{4\pi}\sigma^r = -\frac{2\mu}{\hbar^2 k^2} < \psi_0^c \left| \mathrm{Im}\, t_0 \right| \psi_0^c > \tag{13}$$

For the case of N Bose nuclei, to account for a short range nuclear force between two nuclei, we introduce the following Fermi pseudo-potential $V^F(\vec{r})$

$$\mathrm{Im}\, t_0 = \mathrm{Im}\, V^F(\vec{r}) = -\frac{A\hbar}{2}\delta(\vec{r}) \tag{14}$$

where the short-range nuclear-force constant A is determined from Eqs. (12) and (13) to be $A = 2Sr_B/\pi\hbar$.

For deuteron-deuteron (DD) fusion via reactions *D(d,p)T* and *D(d,n)*^{3}*He*, the S-factor is $S = 110$ *KeV-barn.*

Derivation of Fusion Probability and Rates

For N identical Bose nuclei confined in an ion trap, the nucleus-nucleus fusion rate is determined from the trapped ground state wave function ψ as

$$R_t = -\frac{2}{\hbar}\frac{\Sigma_{i<j} < \psi \left| \mathrm{Im}\, t_{ij} \right| \psi >}{< \psi | \psi >} \tag{15}$$

where $\mathrm{Im}\, t_{ij}$ is given by the Fermi potential Eq. (14), $\mathrm{Im}\, t_{ij} = -A\hbar\delta(\vec{r})/2$.

From Eq. (15), we obtain for a single trap

$$R_t = \sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)N n_B \tag{16}$$

where Ω is the probability of the ground state occupation, $\alpha = e^2/\hbar c, n_B = N/<r>^3$ is Bose nuclei density in a trap, and $B = 3Am/8\pi c$ with $A = 2Sr_B/\pi\hbar$

For the case of multiple ion traps (atomic clusters or bubbles), the total ion-trap nuclear fusion rate *R* per unit time and per unit volume, can be written as

$$R = n_t\sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)N n_B \tag{17}$$

where n_t is a trap number density (number of traps per unit volume) and *N* is the average number of Bose nuclei in a trap.

Fusion Reaction Rates

Our final theoretical formula for the nuclear fusion rate R_{trap} for a single trap containing N deuterons is given by

$$\mathbf{R}_{\mathbf{trap}} = \Omega \mathbf{B} \mathbf{N} \omega^2 \qquad (18)$$

$$\omega^2 = \sqrt{\frac{3}{4\pi}}\,\alpha\left(\frac{\hbar \mathbf{c}}{\mathbf{m}}\right)\frac{\mathbf{N}}{\langle \mathbf{r} \rangle^3}$$

where $\langle r \rangle$ is the radius of trap/atomic cluster, $\langle r \rangle = \langle \Psi | r | \Psi \rangle$,

B is given by $B = 3Am/(8\pi\alpha\hbar c)$,

N is the average number of Bose nuclei in a trap/cluster.

A is given by $A = 2Sr_B/(\pi\hbar)$, where $r_B = \hbar^2/(2\mu e^2)$, $\mu = m/2$,

S is the S-factor for the nuclear fusion reaction between two deuterons (for D(D,p)T and D(d,n)^{3}He reactions, S $\approx$ 55 keV-barn)

All constants are known except Ω, which is the probability of the BEC ground state occupation

$$\mathbf{R_{trap} = 4\left(\frac{3}{4\pi}\right)^{3/2} \Omega A \frac{N^2}{D^3_{trap}} \propto \Omega \frac{N^2}{D^3_{trap}}}$$

with $\mathbf{D_{trap}}$ the average diameter of the trap, $\mathbf{D_{trap} = 2\langle r \rangle}$

Total Reaction Rate

The total fusion rate R_t is given by

$$R_t = N_{trap} R_{trap} = \frac{N_D}{N} R_{trap} \propto \quad \Omega \frac{N}{D^3_{trap}} \tag{19}$$

Only one unknown parameter is the probability of the BEC ground-state occupation, Ω.

Predictions and Comparisons

Prediction : Nuclear fusion rate R does not depend on the Gamow factor in contrast to the conventional theory for nuclear fusion in free space.

⊙This could provide explanations for overcoming the Coulomb barrier and for the claimed anomalous effects for low-energy nuclear reactions in metals. → **Observation [1] The Coulomb barrier is suppressed.**

⊙**This is consistent with Dirac's conjecture** ("The Principles of Quantum Mechanics" (second edition), Oxford 1935, Chapter IX, Section 62) that boson creation and annihilation operators can be treated simply as numbers when the ground state occupation number is large. This implies that for large N each charged boson behaves as an independent particle in a common average background potential and the Coulomb interaction between two charged bosons is suppressed.

⊙For a single trap (or metal particle) containing N deuterons, we have

$$\psi_{BEC}\left\{\left(N-2\right)D's+\left(D+D\right)\right\}\rightarrow\psi^*\left\{{}^4He+\left(N-2\right)D's\right\}\quad\left(Q=23.84\text{ MeV}\right)$$

where ψ_{BEC} is the Bose-Einstein condensate ground-state (a coherent quantum state) with N deuterons, and ψ^* are continuum final states.

⊙Excess energy (Q value) is absorbed by the BEC state and shared by (N-2) deuterons and reaction products.

→ Observations [2] Excess heat production and [4] ^{4}He production.

⊙3D fusion (D + D + D) and 4 D fusion are possible, but their fusion rates are expected to be much smaller than that of the 2D fusion.

Total Momentum Conservation

Initial Total Momentum: $\vec{P}_{D^N} \approx 0$

Final Total Momentum:

$$\{6\} \quad \vec{P}_{D^{N-2}\,{}^4He} \approx 0, \quad \langle T_D \rangle \approx \langle T_{{}^4He} \rangle \approx \frac{Q\{6\}}{N}$$

<T> is the average kinetic energy.

•Excess energies (Q) leading to a micro/nano-scale explosion creating a crater/cavity and a hot spot with firework-like tracks.

• Size of a crater/cavity will depend on number of (D + D) fusions occuring simultaneously in BEC states.

→ Observation [7] Production of hot spots and micro-craters.

• ~10 keV (up to 23.8 MeV) deuterons from {6} lose energies by electrons and induce X-rays, γ-rays, and Bremsstrahlung X-rays.

→ Observation [6] Detection of radiations.

Selection Rule for Two Species Case

Mean-Field Theory of A Quantum Many-Particle System

(Hatre-Fock Theory)

We consider a mixture of two different species of positively charged bosons, with N_1 and N_2 particles, charges $Z_1 \geq 0$ and $Z_2 \geq 0$, and rest masses m_1 and m_2, respectively. We assume $\mathbf{V_i(\vec{r}) = m_i \omega_i^2 r^2 / 2.}$

The mean-field energy functional for the two-component system is given by generalization of the one-component case

$$E = \sum_{i=1}^{2} E_i + E_{\text{int}}, \tag{20}$$

where

$$E_i = \int d\vec{r} \frac{\hbar^2}{2m_i} \left| \nabla \psi_i \right|^2,$$

$$E_{\text{int}} = \frac{e^2}{2} \int d\vec{x} d\vec{y} \frac{\left(Z_1 n_1(\vec{x}) + Z_2 n_2(\vec{x})\right)\left(Z_1 n_1(\vec{y}) + Z_2 n_2(\vec{y})\right)}{\left|\vec{x} - \vec{y}\right|}$$

$n_i = \left|\psi_i\right|^2$, is density of specie i, and $\int d\vec{r} n_i(\vec{r}) = N_i.$ (21)

The minimization of the energy functional, Eq. (3), with subsidiary conditions, Eq. (21), leads to the following time-independent mean-field equations.

$$-\frac{\hbar^2}{2m_i}\nabla^2\psi_i(\vec{r})+(V_i+W_i)\psi_i(\vec{r})=\mu_i\psi_i(\vec{r}), \qquad (22)$$

where

$$W_i(\vec{r})=e^2\int d\vec{y}\left[Z_i^2 n_i^2(\vec{y})+Z_1 Z_2 n_1(\vec{y})n_2(\vec{y})\right]/\left(|\vec{r}-\vec{y}|n_i(\vec{y})\right), \qquad (23)$$

and μ_i are the chemical potentials, $\mu_i=\frac{\partial E}{\partial N_i}$. **(general thermodynamics identity).**

In the Thomas-Fermi (TF) approximation (neglects the kinetic energy terms in Eq. (22)), Eq. (22) reduce to

$$\mu_i=V_i+W_i \qquad (24)$$

which leads to the selection rule (derivation in a backup slide), $\frac{\mathbf{Z_1}}{\mathbf{m_1}}=\frac{\mathbf{Z_2}}{\mathbf{m_2}}$

Application of the Section Rule $\frac{Z_1}{m_1} = \frac{Z_2}{m_2}$ Selection Rule

(m is mass number approximately given in units of the nucleon mass)

$$\frac{\mathbf{Z_1(D)}}{\mathbf{m_1(D)}} = \frac{\mathbf{1}}{\mathbf{2}}, \quad \left(\frac{\mathbf{Z_2(p)}}{\mathbf{m_2(p)}} = \mathbf{1}, \quad \frac{\mathbf{Z_2(T)}}{\mathbf{m_2(T)}} = \frac{\mathbf{1}}{\mathbf{3}} \right), \quad \left(\frac{\mathbf{Z_2(n)}}{\mathbf{m_2(n)}} = \mathbf{0}, \quad \frac{\mathbf{Z_2(^3He)}}{\mathbf{m_2(^3He)}} = \frac{\mathbf{2}}{\mathbf{3}} \right)$$

•Reactions {4} and {5} are forbidden/suppressed→ reaction rates are small

{4} D(m) + D(m) → p(m) + T(m) + 4.03 MeV (m)

{5} D(m)+D(m) → n(m) + ^{3}He(m) + 3.27 MeV (m)

$$\frac{\mathbf{Z_1}}{\mathbf{m_1}}\frac{\mathbf{(D)}}{\mathbf{(D)}} = \frac{\mathbf{Z_2(^4He)}}{\mathbf{m_2(^4He)}} = \frac{\mathbf{1}}{\mathbf{2}}$$

•Reaction {6} is allowed → reaction rate is large

{6} D(m)+D(m)→^{4}He(m) + 23.8 MeV (m)

→ This explains Observation [5] R(4) << R(6) and R(5) << R(6).

{6} D(m) + D(m) $\rightarrow$ ^{4}He(m) + 23.8 MeV (m)

Further Predictions/Comparisons

Effects due to deuterons

Deuterons with Q{6}/N ≈ ~10 keV (up to 23.8 MeV) energy from reaction {6} can interact with surrounding deuterons via the following reaction {7}.

{7} D + D + D $\rightarrow$ n + p + ^{4}He + 21.6 MeV

Reaction {7} may also occur from the BEC state (3D fusion)

Reaction {7} will produce neutrons with a maximum kinetic energy of ~ 18 MeV.

$\{1\}$ $D + D \rightarrow p + T + 4.03$ MeV

$\{4\}$ $D(m) + D(m) \rightarrow p(m) + T(m) + 4.03$ MeV(m)

Further Predictions/Comparisons (continued)

Tritons produced by reaction {1} (1.01 MeV triton) and by reaction {4} (average kinetic energy of Q{4}/N ≈ ~keV) will interact with surrounding deuterons via the following reaction

$\{8\}$ $T + D \rightarrow {}^{4}He(3.52 \text{ MeV}) + n\ (14.07 \text{ MeV})$.

Because of triton kinetic energies of 1.01 MeV or ~ keV, the reaction rate for {8} could be large enough to produce 14.07 MeV neutrons.

{2} $D + D \rightarrow n + {}^{3}He + 3.27$ MeV

{5} $D(m) + D(m) \rightarrow n(m) + {}^{3}He(m) + 3.27$ MeV(m)

{7} $D + D + D \rightarrow n + p + {}^{4}He + 21.6$ MeV

{8} $T + D \rightarrow {}^{4}He(3.52$ MeV$) + n\ (14.07$ MeV$)$.

Effects due to neutrons

Neutrons with 2.45 MeV kinetic energy from reaction {2}, neutrons with Q{5}/N ≈ ~ keV kinetic energy from reaction {5}, or neutrons from reactions {7} and {8} can undergo further reactions, {9}, {10}, and/or {11} below:

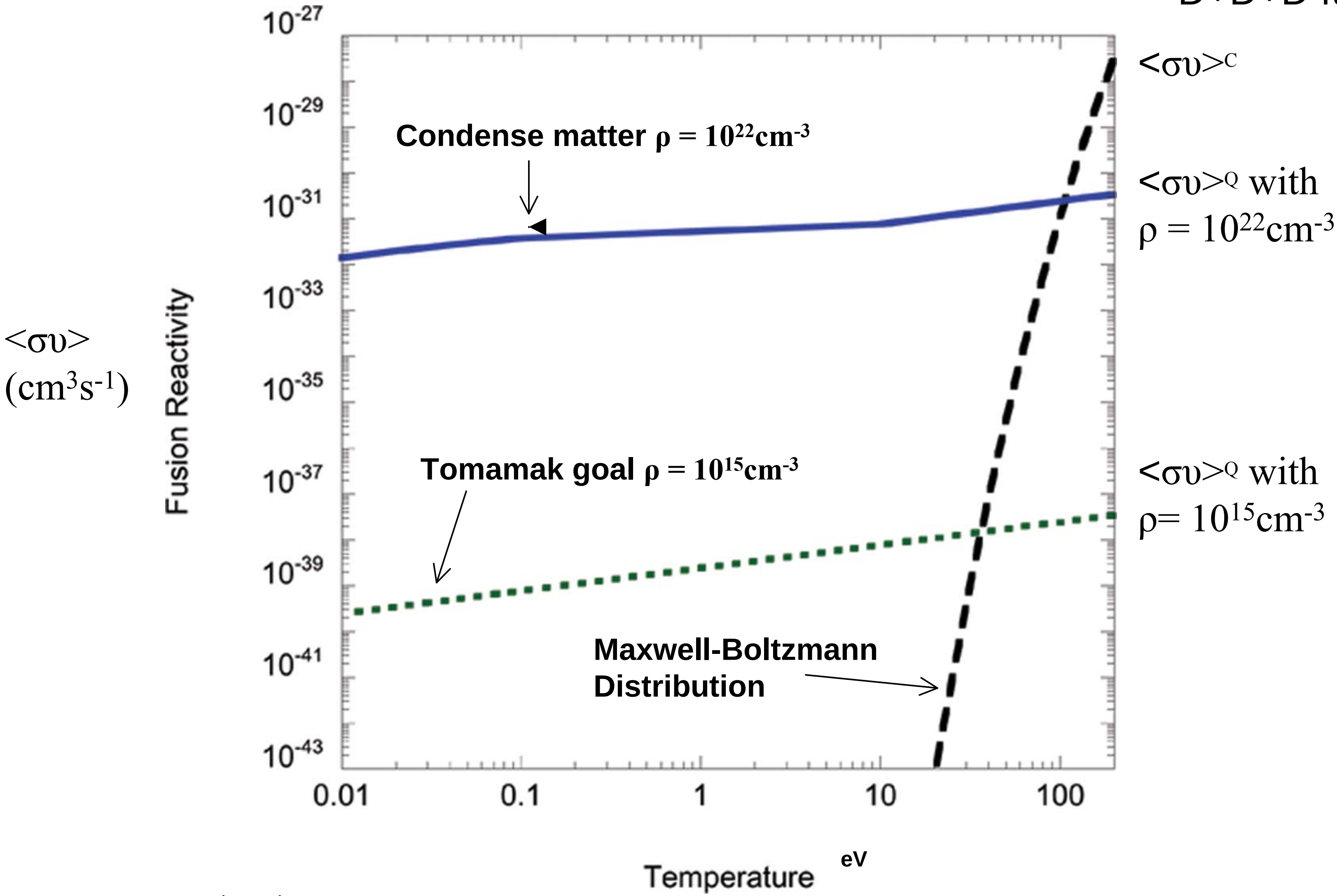
{9} $n + D \rightarrow T + \gamma + 6.26$ MeV

{10} Neutron induced transmutation with other nuclei.

{11} ${}^{12}C(n,n')3\ {}^{4}He$ and ${}^{12}C(n,n'\alpha){}^{8}Be({}^{8}Be \rightarrow 2{}^{4}He)$,

which may have been recently observed by Mosier-Boss et al. (2008) (US Navy SSC-Pacific)

Additional Mechanism based on Quantum Statistical Physics for D+D and D+D+D fusions



Fusion reactivity, $\langle\sigma\upsilon\rangle$ of $D(d,p)^3H$ and $D(d,n)^3He$ reactions in units of cm^3/sec as a function of temperature kT in units of eV. The dashed line is for MB velocity distribution ($\sim e^{E/kT}$). The solid line and the dotted line are for generalized velocity distribution ($\sim E^{-4}$), based on quantum statistical physics (YE Kim and AL Zubarev, Jpn J Appl Phys 45 (2006) L452-L455).

- **Heat after Death (Observation [3])**

  Because of mobility of deuterons in Pd nanoparticle traps, a system of $\sim 10^{22}$ deuterons contained in $\sim 10^{18}$ Pd nanoparticle traps is a dynamical system (in 3 g of 5 nm Pd nanoparticles).

  BEC states are continuously attained in a small fraction of the $\sim 10^{18}$ Pd particle traps and undergo BEC fusion processes, until the formation of the BEC state ceases.

- **Deuteron Mobility Requirement (Observation [8])**

  D/Pd $\geq$ ~0.9 is required for sustaining deuteron mobility in Pd.

  Electric current or pressure gradient is required.

- **Deuterium Purity Requirement (Observation [9])**

  Because of violation of the selection rule,

  presence of hydrogens in deuteriums will surpress the formation of the BEC states, thus diminishing the fusion rate due to the BEC mechanism.

BEC Mechanism on Reactions {4} and {5}

- R{4} << R{6}, R{5} << R{6}, due the selection rule

$$\{4\} \quad D(m) + D(m) \rightarrow p(m) + {}^3He(m) + 4.03MeV(m)$$

$$\{5\} \quad D(m) + D(m) \rightarrow n(m) + T(m) + 3.27MeV(m)$$

$$\{6\} \quad D(m) + D(m) \rightarrow {}^4He(m) + 23.8MeV(m)$$

where n(m) is at energies ~keV.

- ~keV neutron(m) from Reaction {5} can undergo further reactions, {12}, and/or {13} below:

{12} n(m) + D(m) (in BEC State) → T(m) +6.26 MeV(m)

{13} n(m) + D → T + γ + 6.26 MeV

→ Reactions {12} and {13} produce more tritiums than neutrons, R(T) > R(n).

→ R(T) > R(^{3}He)

→This explains Observation [10] more tritium is produced than neutron.

Experimental Tests of Predictions of BECNF theory

- Tests based on the average size of metal particles
- Tests based on the temperature dependence
- Tests for reaction products and heat after death
- Tests for scalability

etc.

→ These experimental tests are needed to

(1) improve and refine the theory and

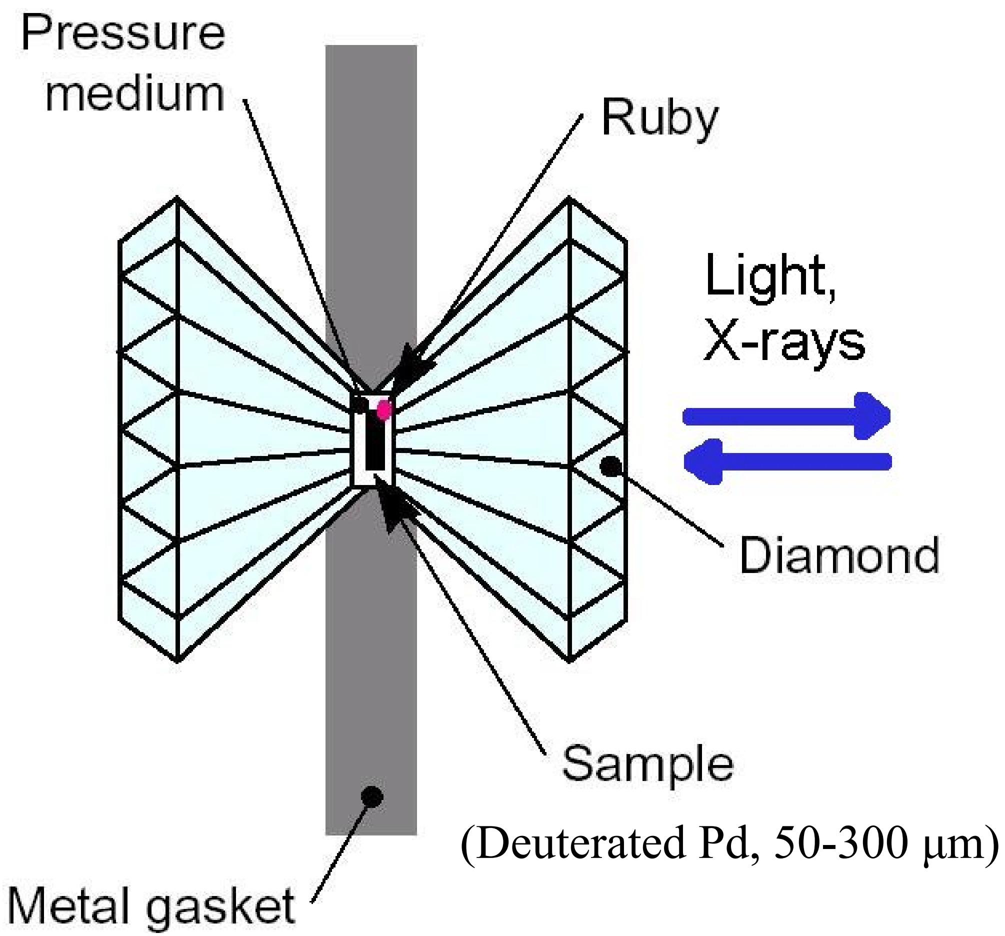
(2) to establish possible protocols for achieving 100 % reproducibility for experimental results, which is needed for possible practical applications.

Basic Fundamental Tests of "Nuclear" Bose-Einstein Condensation

(next slide)

Proposed Basic Fundamental Tests of “Nuclear” BEC

⊙Ultrahigh pressure experiment



Schematics of the core of a diamond anvil cell. The diamond size is a few millimeters at most

- Apply electric current through the sample.
- Sudden change in the resistivity is expected when deuterons form a BEC state at some pressure.
- Emission of radiations and neutrons are expected when BECNF occurs.
- Possibility of using laser beam to measure Raman scattering frequency shifts.

Conclusion and Summary

- Theory of the BEC mechanism provides a consistent conventional theoretical description of the results of many experimental works started by Fleischmann and Pons in 1989 and by many others since then.
- Experimental tests for a set of key theoretical predictions are proposed including scalabilities of the observed effect.
- We need to carry out basic fundamental experimental tests of the concept of "nuclear" Bose-Einstein condensation.
- We also need to carry out experimental tests of the predictions of the BECNF theory in order

  (1) to improve the theory, and also

  (2) to establish possible protocols for achieving 100 % reproducibility for experimental results and for possible practical applications.

Backup Slides

Example with 3g of 50 $\mathring{A}$ Pd particles

- Total number of Pd atoms in 3g, N_{Pd} = 1.7 x 10^{22} Pd atoms

  N_{Pd} = 3g × (6.02 × 10^{23})/106.4g ≈ 1.7 × 10^{22} Pd atoms

  For $N_D \approx N_{Pd}$, $N_D = 1.7 \times 10^{22}$ D atoms

- The number density of Pd, $n_{Pd} \cong 6.8 \times 10^{22}\, cm^{-3} = n_D \leftarrow$----

  n_{Pd} = 12.03 g cm^{-3} × (6.02 × 10^{23})/106..4 g ≈6.8 × $10^{22}cm^{-3}$

- One Pd particle of diameter ~ 50 $\mathring{A}$ contains $N = n_D \left(\frac{\pi}{6}\right)\left(50\,\mathring{A}\right)^3 \leftarrow\!\!\approx 4450$ deuterons

- In 3g of Pd particles, the total number of Pd particle traps is

$$\frac{N_D}{N} = \frac{N_{Pd}}{N} = \frac{1.7 \times 10^{22}}{4.45 \times 10^{3}} \approx 3.8 \times 10^{18} \leftarrow\text{---}$$

particle traps

- For comparison, ~ 3000 atoms are trapped for the atomic case.

Selection Rule

For the BEC mechanism for LENR, we obtain nuclear charge-mass selection rule (approximate).

Nuclear mass-charge selection rule: $\mu_i = V_i + W_i$

We can obtain from Eq. (24) that

$$\mu_2 - \frac{Z_2}{Z_1}\mu_1 = \left(\frac{m_2\omega_2^2}{m_1\omega_1^2} - \frac{Z_2}{Z_1}\right)\frac{m_1\omega_1^2}{2}r^2 .$$

Since μ_i are independent of r, we have proved that Eq. (23) has non-trivial solution if and only if

$$\left(\frac{m_2\omega_2^2}{m_1\omega_1^2} - \frac{Z_2}{Z_1}\right) = 0, \quad \text{or} \qquad \lambda = \frac{m_2\omega_2^2 Z_1}{m_1\omega_1^2 Z_2} = 1. \tag{25}$$

If we assume $\omega_1=\omega_2$, $E_1(\text{G.S.}) = \frac{3\hbar\omega_1}{2} = E_2(\text{G.S.}) = \frac{3\hbar\omega_2}{2}$,

and we have from Eq. (25), $\lambda=m_2Z_1/m_1Z_2=1$ or

$$\frac{Z_1}{m_1} = \frac{Z_2}{m_2} \tag{26}$$

Experimental Tests and Scalability

1. Tests based on the average size of metal nanoparticles

- The fusion rate for a single trap is proportional to

  where N is the number of deuterons in a trap and Ω is the probability of the BEC state occupation.

- For the total fusion rate R_t, we have $$R_{trap} \propto \Omega N^2 / D_{trap}^3$$

where N_{trap} is the total number of traps and N_D is the total number of deuterons.

- Therefore, we obtain $$R_t = N_{trap} R_{trap} \propto \left(\frac{N_D}{N}\right) \Omega N^2 / D_{trap}^3 = N_D \Omega N / D_{trap}^3$$

$$R_t \propto N_D / N D_{trap}^3 \quad \text{for} \quad \Omega \propto \frac{1}{N^2}$$

where D_{trap} is the average trap diameter.

$$R_t(D_{trap}) \propto \frac{1}{D_{trap}^6} \quad \text{for} \quad N = n_D \frac{\pi}{6} D_{trap}^3$$

$$\frac{R_t\left(75\,\mathring{A}\right)}{R_t\left(50\,\mathring{A}\right)} \approx 0.088, \quad \frac{R_t\left(100\,\mathring{A}\right)}{R_t\left(50\,\mathring{A}\right)} \approx 0.016$$

2. Tests based on the temperature dependence

- Since the probability Ω of the BEC ground state occupation increases at lower temperatures ($\Omega \approx 1$ near $T \approx 0$), the total fusion rate R_t will increase at lower temperatures.
  $\rightarrow$ $R_t(T_{low}) > R_t(T_{high})$

- However, R_t is proportional to N_D where N_D is the total number of mobile deuterons. Since $N_D(T_{low}) < N_D(T_{high})$, we expect $R_t\,[N_D(T_{low})] < R_t\,[N_D(T_{high})]$.

- The above opposite temperature dependences for R_t will complicate analysis of the temperature dependence of R_t .

3. Tests for reaction products and heat after death

Examples:

Look for micro/nano-scale craters, cavities, hot spots with firework-like tracks centered at Pd nanoparticle sites, before and after death.

Measure neutron energies to check the reaction products carries average kinetic energies of $\frac{Q\{5\}}{N} \approx \quad \sim keV$

4. Tests for scalability

$$R_t \propto N_{trap} R_{trap} \propto N_{trap} \text{ for the same } R_{trap}$$

$$\rightarrow \quad \frac{R_t(\text{30g Pd particles})}{R_t(\text{3g Pd particles})} = \quad 10, \text{etc.}$$