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Ultra Low-Energy Nuclear Fusion of Bose Nuclei in Nano-Scale Ion Traps

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Abstract: Our recent theoretical investigation on nuclear fusion of integer spin nuclei confined in an isotropic ion trap is described. Solutions of the ground state for charged bosons trapped in the isotropic harmonic oscillator potential are calculated using the equivalent linear two-body method for many-body problems, which is based on an approximate reduction of the many-body Schrödinger equation by the use of a variational principle. Using the ground state wave function, we derive theoretical formulae for rates of nuclear fusion for Bose nuclei confined in ion traps. Our formulae show that the fusion rate does not depend on the Coulomb barrier penetration probability but instead depends on the probability of the ground-state occupation, which is expected to increase as the temperature decreases. Numerical estimates for deuteron-deuteron fusion rates are presented for the case of deuterons trapped in metal powders. Experiments for proof of the principle are suggested to test our theoretical predictions.

1. Introduction

Since the 1989 announcements [1,2] of nuclear fusion at room temperature in D_2 loaded palladium (Pd) and titanium (Ti) cathodes in electrolytic cells containing heavy water (D_2O), there have been persistent claims of observing cold fusion (CF) phenomena. However, most of the reported experimental results are not reproducible. A few notable exceptions are the recently published work by Arata and Zhang [3-5], Bush et al. [6-8], Miles and Bush [9], Case [10], and Stringham *et al.* [11]. We suggested [12] that CF phenomena may be due to the Bose-Einstein condensation of deuterons.

In this paper, we investigate theoretically different aspects of the properties of identical integer-spin nuclei ("Bose" nuclei) confined in ion traps by approximating the ion trap with an isotropic harmonic oscillator potential for simplicity. We report the results of our theoretical investigation on the feasibility of nuclear fusion in such setups ("ion-trap nuclear fusion") using the recently developed equivalent linear two-body (ELTB) method for many-body problems [13]. The ELTB method is based on an approximate reduction of the many-body Schrödinger equation by the use of a variational principle.

2. Anomalous Ultra Low-Energy Nuclear Fusion

Recently, Arata and Zhang [3-5] observed anomalous production of both heat and helium-4 (^4He) from their electrolysis experiments. A Pd metal cylinder containing Pd fine particles was used as a cathode in the electrolysis of heavy water (D_2O). No other nuclear ashes or radiation were observed. The anomalous heat was not observed in the electrolysis of water (H_2O) in the control experiment [3].

A similar anomalous result, producing both heat and ^{4}He but no nuclear ashes nor radiation was also observed by Bush et al. [6-8], Miles and Bush [9], and Case [10]. In Case's experiment [10], gaseous D_2 (at 1-3 atm) was introduced in a catalysis container consisting of activated carbon coated with a platinum-group metal (Pd, Pt, Ir, and Rh), operating at 130-300°C.

The experimental results [3-11] suggest the following radiationless nuclear reaction,

$$d + d \rightarrow {}^4He, (Q = 23.848 MeV), \tag{1}$$

as the explanation of CF. However this reaction cannot occur in free space since it violates momentum conservation. However, the above reaction (1) can occur if it takes place in a many-body quantum state consisting of many deuterons (Bose-Einstein condensation [12] "coherent quantum (CQ) state" [14,15]) because the CQ state can absorb the recoil momentum thus satisfying momentum conservation.

3. Ground-State Solution

In this section, we consider N identical charged Bose nuclei confined in an ion trap. For simplicity, we assume an isotropic harmonic potential for the ion trap to obtain order of magnitude estimates of fusion reaction rates. The Hamiltonian for the system is then

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \frac{1}{2}m\omega^2\sum_{i=1}^{N}r_i^2 + \sum_{i<j}\frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{2}$$

where m is the rest mass of the nucleus. In order to obtain the ground-state solution, we will use the recently developed method of equivalent linear two-body (ELTB) equations for many-body systems [13,14].

For the ground-state wave function Ψ, we use the following approximation [14]

$$\Psi(\vec{r}_1, ...\vec{r}_N) \approx \bar{\Psi}(\rho) = \frac{\Phi(\rho)}{\rho^{(3N-1)/2}}, \tag{3}$$

where

$$\rho = \left[\sum_{i=1}^{N} r_i^2\right]^{1/2}. \tag{4}$$

In reference [13] it has been shown that approximation (3) yields good results for the case of large N.

By requiring that $\bar{\Psi}$ must satisfy a variational principle $\delta\int\bar{\Psi}^* H\bar{\Psi}d\tau = 0$ with a subsidiary condition $\int\bar{\Psi}^*\bar{\Psi}d\tau = 1$, we obtain the following Schrödinger equation for the ground state wave function $\Phi(\rho)$

$$[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + \frac{m}{2}\omega^2\rho^2 + \frac{\hbar^2}{2m}\frac{(3N-1)(3N-3)}{4\rho^2} + V(\rho)]\Phi = E\Phi, \tag{5}$$

where [13]

$$V(\rho) = \frac{N(N-1)}{\sqrt{2}\pi}\frac{\Gamma(3N/2)}{\Gamma(3N/2 - 3/2)}\frac{1}{\rho^3}\int_0^{\sqrt{2}\rho} V_{int}(r)\left(1 - \frac{r^2}{2\rho^2}\right)^{(3N/2-5/2)} r^2\, dr \tag{6}$$

For $V_{int}(r) = e^2/r$, $V(\rho)$ reduces to [13]

$$V(\rho) = \frac{2N\Gamma(3N/2)}{3\sqrt{2\pi}\Gamma(3N/2 - 3/2)}\frac{e^2}{\rho}. \tag{7}$$

Instead of the variable ρ in the Schrödinger equation (12), we introduce a new quantity $\bar{\rho}$ defined as

$$\bar{\rho} = \sqrt{\frac{m\omega}{\hbar}}\rho. \tag{8}$$

Substitution of Eq. (7) into Eq. (5) leads to the following equation

$$\frac{\hbar\omega}{2}[-\frac{d^2}{d\bar{\rho}^2} + \bar{\rho}^2 + \frac{(3N-1)(3N-3)}{4\bar{\rho}^2} + \frac{\bar{\gamma}}{\bar{\rho}}]\Phi = E\Phi, \tag{9}$$

where

$$\bar{\gamma} = \alpha\sqrt{\frac{mc^2}{\hbar\omega}}\frac{4N\Gamma(3N/2)}{3\sqrt{2\pi}\Gamma(3N/2-3/2)} \tag{10}$$

with $\alpha = e^2/(\hbar c) \approx 1/137$. The ground state solution of Eq.(9) has been obtained in the following form

$$\Phi(\bar{\rho}) = \sum_i c_i \bar{\rho}^{\frac{3N-1}{2}} e^{-(\bar{\rho}/\alpha_i)^2/2}, \tag{11}$$

where c_i are determined from Eq.(9) [14].

3. Imaginary Part of the Fermi Potential for Nuclear Interaction

In order to parametrize the short-range nuclear force, we use the optical theorem formulation of nuclear fusion reactions [16]. The total elastic nucleus-nucleus amplitude can be written as

$$f(\theta) = f^c(\theta) + \bar{f}(\theta), \tag{12}$$

where $f^c(\theta)$ is the Coulomb amplitude and $\bar{f}(\theta)$ can be expanded in partial waves

$$\bar{f}(\theta) = \sum_l (2l+1)e^{2i\delta_l^c} f_l^{n(el)} P_l(\cos\theta). \tag{13}$$

In Eq. (13), δ_l^c is the Coulomb phase shift, $f_l^{n(el)} = (S_l^n - 1)/2ik$, and S_l^n is the l-th partial S-matrix for the nuclear part. For low energy we can write [16]

$$Im f_l^{n(el)} \approx \frac{k}{4\pi}\sigma_l^r,$$

where σ_l^r is the partial wave reaction cross section. For the dominant contribution of only s-wave, we have

$$Im f_0^{n(el)} \approx \frac{k}{4\pi}\sigma^r, \tag{14}$$

where σ^r is conventionally parameterized as

$$\sigma^r = \frac{S}{E}e^{-2\pi\eta}, \tag{15}$$

$\eta = \frac{1}{2kr_B}, r_B = \frac{\hbar^2}{2\mu e^2}, \mu = m/2$, and S is the S-factor for the nuclear fusion reaction between two nuclei.

In terms of the partial wave t-matrix, the elastic scattering amplitude, $f_l^{n(el)}$, can be written as [16]

$$f_l^{n(el)} = -\frac{2\mu}{\hbar^2 k^2} < \psi_l^c \mid t_l \mid \psi_l^c >, \tag{16}$$

where ψ_l^c is the Coulomb wave function.

Introducing a new quantity U as the imaginary part of t_0,

$$U = Im(t_0), \tag{17}$$

we have

$$\frac{k}{4\pi}\sigma^r = -\frac{2\mu}{\hbar^2 k^2} < \psi_0^c \mid U \mid \psi_0^c > . \tag{18}$$

For our case of N Bose nuclei, to account for a short range nature of nuclear forces between two nuclei, we introduce the following Fermi pseudo-potential $V^F(\vec{r})$,

$$ImV^F(\vec{r}) = -\frac{A\hbar}{2}\delta(\vec{r}), \tag{19}$$

where the short-range nuclear force constant A is determined from Eqs.(15) and (18) to be [14]

$$A = \frac{2Sr_B}{\pi\hbar}. \tag{20}$$

We note that Eq.(20) above relates the short-range nuclear force constant A to experimentally extracted value of the S-factor S in Eq.(15). The S-factor is a nuclear force strength factor (or coupling constant) independent of Coulomb force contribution which is parameterized by the Gamow factor $e^{-2\pi\eta}$ in Eq.(15). The values of the S-factor have been extracted from low-energy nuclear reaction cross-section using Eq.(15).

For a many Boson (deuteron) system, the two-body Coulomb wave function ψ_o^c in Eq.(18) is no longer applicable, and hence the Gamow factor $e^{-2\pi\eta}$ is not also applicable. Therefore we must now use an eigenstate solution Ψ (or Φ) of the many-body Hamiltonian, Eq.(2), which includes all pair-wise Coulomb interactions between deuterons. In the following section, we derive the fusion rates for many deuterons confined in a trap.

4. Fusion Rates

For N identical Bose nuclei confined in an ion trap, the nucleus-nucleus fusion rate is determined from the trapped ground state wave function Ψ as

$$\tilde{R} = -\frac{2}{\hbar}\frac{\Sigma_{i<j} < \Psi|ImV_{ij}^F|\Psi >}{< \Psi|\Psi >}, \tag{21}$$

where ImV_{ij}^F is the imaginary part of the Fermi potential given by Eq.(19).

The substitution of Eq.(3) into Eq.(21) yields [14]

$$\tilde{R} = \frac{AN(N-1)\Gamma(3N/2)}{2(2\pi)^{3/2}\Gamma(3N/2-3/2)}\frac{\int_0^\infty \Phi^2(\rho)\frac{1}{\rho^3}d\rho}{\int_0^\infty \Phi^2(\rho)d\rho}. \tag{22}$$

For large N, we use an approximate solution for $\Phi(\rho)$ (see Eq.(11))

$$\Phi(\rho) \approx \tilde{\rho}^{\frac{3N-1}{2}} e^{-(\tilde{\rho}/\alpha_t)^2/2} \tag{23}$$

where $\alpha_t = (\zeta/3)^{1/3}, \zeta \approx 2\sqrt{mc^2/2\pi\hbar\omega}\alpha N$, and $\tilde{\rho} = \sqrt{m\omega/\hbar}\rho$.
Using Eq.(23), we obtain from Eq.(22)

$$R_t = \Omega BN\omega^2, \tag{24}$$

where

$$B = \frac{3A}{8\pi\alpha}(\frac{m}{\hbar c}), \tag{25}$$

and Ω is the probability of the ground state occupation,

The average size $<r>$ of the ground-state for Bose nuclei confined in a trap can be calculated using the ground-state wavefunction, Eq.(23), and is related to ω by the following relation for the case of large N,

$$\omega^2 = \sqrt{\frac{3}{4\pi}}\alpha\left(\frac{\hbar c}{m}\right)n_B, \tag{26}$$

where $\alpha = e^2/\hbar c$. and $n_B = N/ < r >^3$ is Bose nuclei density in traps. In terms of n_B we can rewrite R_t, Eq.(24), as

$$R_t = \sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)Nn_B \ . \tag{27}$$

For the case of multiple ion traps in a metal with each trap containing N Bose nuclei, we define a trap number density n_t (number of traps per unit volume) as $n_t = n_B/N =< r >^{-3}$. where N is the average number of Bose nuclei in a trap. For this case, the total ion-trap nuclear fusion rate R per unit time and per unit volume can be written as

$$R = \sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)n_t Nn_B \ . \tag{28}$$

We note a very important fact that both R_t and R do not depend on the Gamow factor in contrast to the conventional theory for nuclear fusion in free space. This is consistent with the conjecture noted by Dirac [17] and Bogolubov [18] that boson creation and annihilation operators can be treated simply as numbers when the ground state occupation number is large. This implies that for large N each charged boson behaves as an independent particle in a common average background potential and the Coulomb interaction between two charged bosons is suppressed. Furthermore, the rates R_t and R are proportional to Ω which is expected to increase as the operating temperature decreases. Since n_B is nearly constant and $n_t = n_B N^{-1} = <r>^{-3}$ in Eq.(28), and the average distance between nearest two deuterons is $n_D^{-1/3} \approx 2.45$Å, the nuclear fusion rate per unit volume, R, given by Eq.(28) is proportional to n_B^2. This implies that R can be made larger by increasing the deuteron density. Since metal powders or blacks can provide larger total surface area, they are more desirable and efficient systems for achieving steady high density states of deuterons in cluster-traps than bulk metals. This may explain why Arata and Zhang observe a larger effect and can reproduce their former results consistently using Pd blacks (powders) of $<r>\approx$ 40nm while others have experienced difficulties in attaining reproducible results. However, N decreases as $<r>$ decreases, and the CQ state may not be achieved with a much smaller value of N. Furthermore, Eq.(28) also becomes inaccurate as N becomes smaller.

5. Application to Deuteron-Deuteron Fusion

5.1 Multiple-Trap Experiment

For deuteron-deuteron (DD) fusion via reactions $D(d,p)T$ and $D(d,n)^3He$ the S-factor is $S = 110keV$-$barn$ and hence we find from Eq.(20) the nuclear rate constant to be

$$A \approx 1.5 \times 10^{-16} cm^3/sec, \tag{29}$$

and from Eqs.(25) and (29), we have

$$B = 2.6 \times 10^{-22} sec. \tag{30}$$

In order to apply our theoretical results for R_t and R, Eqs.(27) and (28), to the results of Arata and Zhang [3-5], we assume a transient situation in which a constant net (in and out) flux of deuterons is maintained across the surfaces of each Pd atom cluster so that deuterons inside the cluster are mobile while maintaining a constant density of $n_D \approx n_{Pd}$, i.e., each Pd atom cluster acts as an effective trap for deuterium ions ("cluster-trap").

For the experiments by Arata and Zhang [3-5], $3g$ of Pd blacks (powders) consisting of $\sim$ 40 nm average size clusters was used. $3g$ of Pd blacks contain $\sim 1.7 \times 10^{22}$Pd atoms ($\approx 3g \times (6 \times 10^{23})/106.4g$ where 106.4g is a molar weight of Pd). Since the volume density of Pd metal is 12.02 gcm^{-3}, the number density of Pd atoms is $n_{Pd} \approx 6.8 \times 10^{22} cm^{-3}$. Since they achieved a high deuteron density of $n_D \approx n_{Pd}$ ($n_D = n_B$), we have $n_D \approx 6.8 \times 10^{22} cm^{-3}$ and also $N_T \approx 1.7 \times 10^{22}$ D atoms in 3g of Pd blacks. For a cluster of $<r> \approx 40nm (= 4 \times 10^{-6} cm)$ size, it contains $N = n_D <r>^3 \approx 4.35 \times 10^6$ D's. We note that $N <r>^{-3} = n_D$ in Eqs.(27) and (28), and the average distance is $n_D^{-1/3} \approx 2.45$Å between nearest two deuterons.

Using the above numerical values, we have from Eq.(27)

$$R_t = \Omega B(10^{34}) sec^{-2} \text{ per cluster-trap,} \tag{31}$$

where B is given in units of seconds. From Eq.(28) with $n_t = n_D/N = <r>^{-3} = 1.56 \times 10^{16} cm^{-3}$, we obtain for the DD fusion rate

$$R = n_t R_t = \Omega B(1.56 \times 10^{50}) sec^{-2} cm^{-3}, \tag{32}$$

with B given in units of second.

For DD fusion via reactions $D(d,p)T$ and $D(d,n)^3He$, $B = 2.6 \times 10^{-22} sec$ (see Eq.(30)), and hence we obtain from Eq.(31),

$$R_t = \Omega(2.6 \times 10^{12}) sec^{-1} \text{ per cluster-trap,} \tag{33}$$

and from Eq.(32),

$$R = \Omega(4 \times 10^{28}) sec^{-1} cm^{-3}. \tag{34}$$

For reaction (1) occurring in the coherent quantum state (CQS), the cross-section ($\sigma_{CQS}(dd \rightarrow {}^4He)$) for reaction $dd \rightarrow {}^4He$ may be larger than that of reactions, $D(d,p)T$ or $D(d,n)\,{}^3He$. If the effective S-factor for $\sigma_{CQS}(dd \rightarrow {}^4He)$ happens to be larger than those for $\sigma(D(d,p)T)$ and $\sigma(D(d,n)\,{}^3He)$, it can provide a theoretical explanation of the anomalous effect [3-11].

If the predominant production of T over ${}^4He$ or ${}^3He$ as claimed previously by many is definitively confirmed in future experiments, we need to investigate other possible many-body effects and mechanisms such as possible modification of the S-factors in different reaction channels due to many-body nature of the system.

For DD fusion via reaction (1), the experimental results [3-11] indicate that the S-factor for reaction (1) is greater by at least 10^6 than that for reactions $D(d,p)T$ and $D(d,n)^3He$ implying $B = 2.6 \times 10^{-16} sec$ for this reaction. Using this value of B in Eqs.(31) and (32), we obtain for reaction (1),

$$R_t^{^4He} = \Omega(2.6 \times 10^{18}) sec^{-1} \text{ per cluster-trap,} \tag{35}$$

and

$$R^{^4He} = \Omega(4 \times 10^{34}) sec^{-1} cm^{-3}. \tag{36}$$

The reaction rate per deuteron (D) is then

$$\Lambda^{^4He} = R^{^4He}/n_D = \Omega(6 \times 10^{11}) sec^{-1} \text{ per } D \ . \tag{37}$$

The observed excess heat production rate is $\sim 5W$. If the excess heat produced is due to reaction (1), the reaction rate is $R_{AZ}^{^4He} = 1.3 \times 10^{12}$ $(D + D)$ reactions/sec, and hence the reaction rate per D is

$$\Lambda_{AZ}^{^4He} = \frac{R_{AZ}^{^4He}}{N_T} = 0.76 \times 10^{-10} sec^{-1} \text{ per } D, \tag{38}$$

which is comparable to $\Lambda_{FP}^{^4He} \approx 10^{-10} sec^{-1}$ per D originally claimed by Fleischmann et al. [1]. Comparing Eq.(37) with Eq.(38), we infer that

$$\Omega < 10^{-22} \ . \tag{39}$$

Comparison of $\Lambda_{FP}^{^4He} \approx 10^{-10} sec^{-1}$ per D for reaction (1) with $\Lambda_{Jones}^{n} \approx 10^{-23} sec^{-1}$ per D for reaction $D(d,n)^3He$ claimed by Jones et al. [2], suggests that the S-factor for reaction (1) is greater by 10^{13} than that for reactions $D(d,p)T$ and $D(d,n)^3He$ and hence $B = 2.6 \times 10^{-9} sec$. For this case, we have

$$\Omega \approx 10^{-29} \ . \tag{40}$$

We note that $\Omega \approx 10^{-29} \sim 10^{-22}$ is much larger than the Gamow factor at ambient temperatures. Additional experimental measurements are needed to extract a more precise value of Ω.

5.2 Single-Trap Experiment

Recently, we suggested experiments for proof of the principle to test the proposed CQS nuclear fusion mechanism [15] based on the fusion rate R_t, Eq.(27).

In a recent experiment [19], $^9Be^{\dagger}$ ions were confined in a cylindrical Penning trap consisting of an electrostatic quadrupolar potential and a uniform magnetic field. A typical density achieved is $\sim 4 \times 10^8 cm^{-3}$ with $N = 10^6$. If a similar density for deuterons, $n_D = n_B = 4 \times 10^8 cm^{-3}$ with $N = 10^6$ can be achieved in a same type of experiment, the fusion rate R_t, Eq.(27) is

$$R_t \approx \Omega B(1.35 \times 10^{19}) sec^{-2} \ . \tag{41}$$

For reactions, $D(d,n)\ ^3He$ and $D(d,p)T$, we have $B = 2.6 \times 10^{-22} sec$ (see Eq.(30)) and hence the fusion rate $R_t^{n+p}(\Omega)$ is from Eq.(41),

$$R_t^{n+p}(\Omega) \approx \Omega(3.5 \times 10^{-3}) sec^{-1} \ . \tag{42}$$

At room temperatures with $\Omega = 10^{-29}$ (see Eq.(40)), we have

$$R_t^{n+p}(10^{-29}) \approx 3.5 \times 10^{-32} sec^{-1}, \tag{43}$$

which implies that the effect is too small to be detected. On the other hand, if we can achieve $\Omega \approx 10^{-1}$ by cooling deuterons ($nK \sim mK$), then we have

$$R_t^{n+p}(10^{-1}) \approx 3.5 \times 10^{-4} sec^{-1}, \tag{44}$$

which is a detectable rate for neutrons. It is known that one can achieve temperatures of $\sim mK$ for the case of anti-protons by injecting electrons [20].

A more promising case is for reaction (1), $D + D \rightarrow {}^4He$. For this reaction, the inferred value of B is $B = 2.6 \times 10^{-9} sec$, and the fusion rate $R_t^{^4He}$ is from Eq.(41),

$$R_t^{^4He}(\Omega) \approx \Omega(3.5 \times 10^{19}) sec^{-1} \quad . \tag{45}$$

At room temperatures with $\Omega = 10^{-29}$, Eq.(45) yields

$$R_t^{^4He}(10^{-19}) \approx 3.5 \times 10^{-19} sec^{-1}, \tag{46}$$

which is again impossible to detect as in the case of Eq.(43). However, if we can achieve $\Omega \approx 10^{-10}$ by cooling deuterons in a trap with injection of positrons, we have from Eq.(45),

$$R_t^{^4He}(10^{-10}) \approx 3.5 sec^{-1}, \tag{47}$$

which is a detectable rate for ^4He. Therefore, it is worthwhile to carry out ion-trap experments with cooled deuterons as tests of the proof of the principle for our CQS nuclear fusion mechanism.

6. Summary and Conclusions

Using the recently developed theoretical method ("Equivalent Linear Two-Body Method") [13], we have obtained an approximate ground-state solution of many-body Schrödinger equation for a system of N identical charged bosons confined in an isotropic harmonic oscillator potential. The solution is expected to be accurate for large N [13]. The solution is used to obtain theoretical formulae for estimating the probabilities and rates of nuclear fusion for N identical Bose nuclei confined in an ion trap. Our results show that the Coulomb interaction between two charged bosons is suppressed for the large N case. This is consistent with the conjecture made by Dirac [17] and used by Bogolubov [18] that each interacting neutral boson behaves as an independent particle in a common average background for the large N case. The fusion rate formula is applied to deuteron-deuteron fusion rates for deuterons trapped in metal atomic clusters. In the following, we summarize predictions and consequences of our theoretical formulation and Eqs.(27) and (28), for nuclear fusion of Bose nuclei (deuterons) confined in ion traps or trapped in metal powders (clusters).

a. The fusion rate R_t Eq.(27), and R, Eq.(28), does not depend on the Gamow factor, but instead depends on the probability of the ground-state occupation (Ω). Ω is expected to increase as the temperature decreases. This implies that the fusion rate increases as the temperature decreases. The experimental data suggest $\Omega < 10^{-22}$ [3-5] or $\Omega \approx 10^{-29}$ [1,2]. Although there have been some claims of observing higher fusion rates at higher temperatures from electrolysis experiments, they are not yet definitive and conclusive. Our contrary prediction can be tested by future experiments of Arata-Zhang type or single-trap type (described below in (d)).

b. The fusion rate R, Eq.(28), is proportional to n_D^2 where n_D is the deuteron density. Since metal powders or blacks can provide larger total surface area, they are more desirable and efficient systems for achieving steady high density states of deuterons in cluster-traps than bulk metals. This may explain the reproducible results of Arata-Zhang [3-5] who used Pd powders with $<r> \approx 40nm$, while others had difficulties in obtaining reproducible results from electrolysis experiments with heavy water using bulk metal cathodes.

c. Our theoretical formulation is based on deuterons (positive deuterium ions), but is also applicable to deuterion plasma (deuterons and electrons). We need transition metal (such as Pd, Ti, Nb, Zr, etc.) powders which can lower activation energies of D_2O or D_2 and convert them into deuterium ions or atoms efficiently as they make contact with and enter into the metal cluster (trap).

d. As a test of the proof of the principle for our CQS nuclear fusion mechanism, we propose single-trap experiments in which deuterons are confined in an ion trap and cooled by injection of positrons.

More detailed descriptions of some parts of this paper will be published elsewhere [21].

Appendix: Alternative Derivation

In this appendix, we describe an alternative derivation of the ground-state wavefunction and the fusion rate formula, Eqs.(24) and (25).

To describe ground-state properties of the system of the N Coulomb, interacting bosons, we start the following equation for the mean-field theory for bosons

$$[-\frac{\hbar^2}{2m}\Delta + \frac{m\omega^2}{2}r^2 + (N-1)e^2\int\frac{d\vec{r}'}{|\vec{r}-\vec{r}'|}|\bar{\phi}(\vec{r}')|^2]\bar{\phi}(\vec{r}) = \mu\bar{\phi}(\vec{r}), \qquad (A-1)$$

where the chemical potential μ is related to the ground-state energy E and particle number N by the general thermodynamic identity

$$\mu = \frac{\partial E}{\partial N}. \qquad (A-2)$$

For the case of $N >> 1$ and $N\gamma_c >> 1$, where $\gamma_c = 2\sqrt{mc^2/\hbar\omega}$, the ground-state solution of Eq.(A-1) is found to be

$$|\bar{\phi}(\vec{r})|^2 = \frac{3}{4\pi N\gamma_c}\theta((\gamma_c N)^{2/3} - r^2\frac{m\omega}{\hbar})(\frac{m\omega}{\hbar})^{3/2}, \qquad (A-3)$$

where θ denotes the positive unit step function. Straightforward calculations with $|\bar{\phi}(\vec{r})|^2$ from Eq.(A-3) yield

$$\mu = \frac{3}{2}\hbar\omega(\gamma_c N)^{2/3}, E = \frac{9}{10}\hbar\omega(\gamma_c)^{2/3}N^{5/3}. \qquad (A-4)$$

Substitution of Eq.(A-3) into Eq.(21) leads to the previous result for fusion rate given by Eqs.(24) and (25). From Eq.(A-3), we obtain the size d of the ground state for Bose nuclei as $d = \sqrt{\frac{\hbar}{m\omega}}(\gamma_c N)^{1/3}$, which is related to ω by the following relation for the case of large N,

$$\omega^2 = \alpha(\frac{\hbar c}{m})n_B, \qquad (A-5)$$

instead of Eq.(26). For this alternative derivation, all formulae for fusion reaction rates, Eqs.(27), (28), (31-37), (41-47) are to be multiplied by a factor of 2.

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